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How to Sell Jobs*

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Abstract

Profit-maximizing firms should fill job positions at the lowest possible cost. Because employees may have preferences over the attributes of their jobs, we can view this problem as one of finding the optimal way to sell job attributes to potential employees. In this paper, we characterize the optimal mechanism by which a firm can sell jobs with desirable attributes. This mechanism is implemented by offering employees a long-term employment contract in which firms create a number of low-quality job positions and offer them to young employees, while only a subset of these employees are promoted to a desirable job. In contrast to the traditional compensating differentials framework, job desirability and wages are positively related in the optimal contract. Our analysis provides a novel framework for thinking about a number of phenomena, such as the span of control, inequality within and between generations, and the effect of competition on employment and wages.

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1 Introduction

Employees have preferences over the attributes of their jobs. Monetary compensation is just one of such attributes; typical non-pecuniary attributes include job status, prestige, location, benefits of control, flexibility, and work-life balance, among others.[1] We can thus think of firms as entities that sell conventional goods to the market and simultaneously sell non-market goods (i.e., jobs with certain attributes) to their employees (as in Rosen, 1986a).

In a setting in which firms are endowed with jobs with desirable attributes, we develop a theory of optimal employment contracts and job design. Firms have a fixed number of high-quality jobs, which are job positions with attributes that potential employees find attractive. The problem firms face is how to fill these positions at the lowest possible cost. Alternatively, we can think of this problem as one of finding the optimal way to sell desirable job attributes to potential employees.[2] We show that the optimal way to sell highly desirable jobs is by creating new low-quality jobs that serve as a port of entry to the firm. In an optimal contract, firms first assign all young workers to low-quality jobs and then promote only a subset of them to high-quality jobs, as in up-or-out contracts.

Ever since Adam Smith (1776), economists have used the theory of compensating differentials as the main framework for thinking about the problem of selling job attributes (see Rosen (1986a) for a comprehensive review of this topic). According to this theory, with all else held constant, a job with a desirable attribute will command a lower wage. By contrast, in our model, the optimal contract is such that more desirable jobs pay higher wages. Three aspects of our model drive this result: (i) employees perceive positive job attributes and monetary compensation as complements, (ii) firms can create new positions in low-quality jobs (i.e., jobs without desirable attributes), and (iii) firms can write long-term employment contracts.

We model the complementarity between job attributes and monetary compensation as desirable job attributes that positively affect the marginal utility of income. We borrow this assumption from Rosen (1997, 2002). Such preferences often lead to situations in which

[1] For some recent evidence of workers' and executives' willingness to pay for job attributes, see Mas and Pallais (2017) and Focke, Maug, Niessen-Ruenzi (2017).

[2] Here, we think of firms as the sellers of the job. A different but related issue is that of third parties selling access to a job, such as when corrupt government officials make appointments in exchange for bribes (see Weaver, 2017).

agents behave in a risk-seeking manner for intermediate ranges of income, as in Friedman and Savage (1948) and Friedman (1953). The literature on preferences over status similarly assumes that status positively affects the marginal utility of income (Becker, Murphy, and Werning, 2005; Auriol and Renault, 2008; Roussanov, 2010; Ray and Robson, 2012).

Unlike most of the related literature, we allow firms to create new low-quality jobs. From a technological perspective, such jobs may be inefficient, and thus, firms use these jobs only as a means to implement an optimal contract. Because firms have the option to create low-quality jobs, long-term employment contracts arise endogenously as the optimal mechanism for selling jobs.

In an optimal career path, the wage attached to the desirable job is increasing in the desirability of that job. Attaching a high wage to a desirable job allows firms to hire a large number of employees to fill low-quality job positions. Thus, firms profit from long-term career path contracts by hiring many relatively underpaid employees to fill low-quality job positions to compensate for the overpayment of those who are promoted to desirable jobs. Employees are willing to accept such contracts because the marginal utility of income is increasing in job desirability. As job desirability improves, employees are willing to accept a lower probability of being promoted in exchange for a higher wage upon promotion.

Firms endowed with more desirable jobs hire more young employees, which implies that more desirable jobs are associated with wider spans of control. Because in our model the marginal job that is created is always socially inefficient, in equilibrium firms with wider spans of control employ inefficiently high numbers of young employees. That is, industries with prestigious occupations will feature overemployment of young employees.

The model predicts that competition in the labor market affects the shape of optimal career path contracts. We show that an increase in labor supply decreases the hours worked per employee and these employees' probability of promotion. A perhaps unexpected result is that more competition among workers increases the wages paid to those who are promoted to a desirable job. An increase in the supply of workers allows firms to hire more employees who compete for promotions. Thus, from an employee's perspective, the increase in wages upon promotion is offset by the diminishing probability of being promoted. We also show that more competition for jobs increases intra-generation inequality because such competition leads to even higher wages upon promotion. By contrast, competition for jobs decreases

inter-generational inequality for two reasons: both the expected utility of being promoted when old and the number of hours worked while young decrease with greater competition for jobs.

As in Raith (2003), in our model, different sources of competitive pressure in the product market have different effects on employment contracts. An increase in competitive pressure caused by tougher competition among industry incumbents increases the wage upon promotion. By contrast, an increase in competitive pressure caused by lower costs of entering the industry decreases the wage upon promotion.

As is often the case in the personnel economics literature (Oyer and Schaefer, 2011; Lazear and Oyer, 2012), our question (*how to sell jobs*) is a normative one. We show that firms that sell desirable jobs should offer young employees low-quality jobs, which may then lead to highly-paid prestigious jobs. Although testing this implication is beyond the scope of this paper, we note that the model delivers many predictions with empirical content. For example, unlike compensating differential theories, our model does not necessarily predict a negative correlation between wages and the willingness to pay for job attributes in the cross-section. Bonhomme and Jolivet (2009) find strong evidence that workers have preferences for job attributes, but these preferences do not translate into significant compensating differentials in the cross-section.

An important result in our model is the optimality of up-or-out contracts. There are two groups of theories of up-or-out promotion contests. The first is based on moral hazard considerations. In these theories, the problem is typically how to induce employees to take certain non-contractible actions when firms have private information about performance or worker ability (Kahn and Huberman (1988); Prendergast (1992, 1993); Waldman (1990); Ghosh and Waldman (2010)).[3] Another reason why up-or-out contracts can be optimal is that they provide steeper incentives than standard promotion practices (Auriol, Friebel, and von Bieberstein, 2016). The second group of theories is based on selection issues. For example, in O'Flaherty and Siow (1992) and Demougin and Siow (1994) up-or-out rules arise because of the value of the option to learn a new employee's type. Experienced employees who are not qualified enough to be promoted should give way to younger employees for whom

[3]Rebitzer and Talyor (2007) offer an alternative moral hazard theory of up-or-out rules based on the need to prevent experienced employees from leaving the firm and taking key assets (such as clients) with them.

their type is yet to be discovered. The same option-value logic explains the optimality of up-or-out rules in Barlevy and Neal (2017). To the best of our knowledge, our model is the first to rationalize up-or-out rules in a setting without either moral hazard or selection concerns.

Another important result in our model is that young employees work inefficiently long hours. Alternative theories of heavy workloads tend to focus on incentive (e.g., Axelson and Bond, 2015) or selection issues (e.g., Barlevy and Neal, 2017). In our model, working long hours is how employees compensate firms for offering desirable jobs. This setting is similar to Fudenberg and Rayo's (2017) model of accumulation of general human capital. In that model, workers are assigned to both desirable tasks (i.e., tasks that improve workers' skills) and menial tasks simultaneously. The performance of menial tasks compensates the principal for offering general training to workers.

Just like our theory, tournament theory (Lazear and Rosen, 1981; Green and Stokey, 1983; Nalebuff and Stiglitz, 1983; Rosen, 1986b) also implies the optimality of long-term contracts in which only some employees are promoted to a higher-paying job. In tournament theory, promotions are a means of incentivizing workers to exert effort. Prizes and winning probabilities depend on relative performance and are chosen optimally to provide incentives for effort provision at early career stages. Ours is not a theory of tournaments but instead a theory of internal career paths. We show that firms find it optimal to create entry-level jobs that lead to higher-level jobs as a way to extract greater surplus from young employees.

Our paper is also related to the literature on job design. This literature focuses mainly on the question of how to allocate existing tasks or jobs to different agents in the organization (e.g., Itoh, 1994; Hemmer, 1995; Prendergast, 1995; Zabojnik, 2002). In our model, job design serves a different purpose: the creation of new tasks or jobs improves firms' ability to extract rents from their employees. Prendergast (1995) develops a theory of inefficient job assignments based on firms' inability to sell jobs. In his model, as in ours, employees have preferences over jobs with desirable attributes but firms cannot extract rents from employees through monetary transfers. An important difference in our model is that firms can design new jobs as a means of extracting rents from employees.

In a related paper, Ke, Lin and Powell (2018) develop a dynamic moral hazard model in which firms can create new jobs both at the bottom and at the top of promotion hierarchies.

In their model, it is optimal for firms to create new and potentially inefficient jobs at the top of hierarchies. Thus, the optimal span of control is inefficiently low relative to what would be justified by productive efficiency alone. In contrast, in our model, optimal contracts require the creation of inefficient jobs at the bottom of the hierarchy.

We take as given that a job attribute is concentrated in few jobs, and we then endogenize the number of jobs without the desirable attribute. Our model thus differs from that of Auriol and Renault (2008), who consider a different job design problem in which the firm allocates a fixed amount of status to a fixed number of jobs in the organization. They show that because of the complementarity between status and income, it is optimal to concentrate status and wages in only a few jobs in the organization as a means to provide incentives for noncontractible effort.

2 The Model

2.1 Setup

Agents live for two periods: young age and old age. In each period, a continuum of mass E of young agents enters the labor market. There is a mass F (which we call a sector) of infinitely lived firms. We assume that $F < E$. Each firm in sector F has a technology that requires a continuum of mass 1 of employees (either young or old) working in job h to generate revenue $R > 0$. The technology is not scalable.

Agents can either work for a firm in sector F or work in the alternative sector, which can also be interpreted as self-employment. In the alternative sector, young agents receive wage $\underline{w}^y$ and old agents receive $\underline{w}^o$; without loss of generality, we assume that young and old agents both have the same outside wage: $\underline{w}^y = \underline{w}^o = \underline{w}$. We refer to jobs in the alternative sector as *outside jobs*.

All jobs require a minimum amount of hours ε. Total hours worked on a job is given by $\varepsilon + e$; we refer to e as overtime. For notational simplicity only, we normalize $\varepsilon = 0$. We assume that overtime in job h or in the outside job generates no additional revenue to firms.

2.2 Preferences

Agents have preferences over income and jobs attributes according to a utility function $U(c, q)$, where c is consumption (i.e., income) and q is the job attribute. More desirable jobs have higher values of q. We also assume

$$\frac{\partial^2 U(c,q)}{\partial c \partial q} > 0.$$

To understand this assumption, consider the following example, which is taken from Rosen (1997, 2002). There are two jobs: Job 1, which is located in a small town, and Job 2, which is located in a big city. Assume that all else constant, agents prefer the amenities of the big city, that is, $q_2 > q_1$. In addition, there are many more opportunities to spend money in a big city because some goods and services may be available only to big city dwellers (i.e., restaurants, theaters, etc.). Thus, for the same level of income in both locations, the value of an extra dollar is higher in the big city.

The desirable job attribute may be status instead of location. For example, Becker, Murphy, and Werning (2005) argue that "*higher status raises the marginal utility of a given level of income partly because persons with high status often have access to clubs, friends, and other "goods" that are costly but are not available to those with low status*" (p. 284). Similarly, higher income increases the marginal utility of status, as argued by Auriol and Renault (2008): "*richer agents care more about their status in the sense that they are willing to exert more effort in order to improve it*" (p. 310).

In our model, we do not take a stand regarding what the job attribute is; it could be location, status, or any other desirable attribute that is complementary to income.

We assume that workers care not only about consumption and job attributes but also about the number of hours worked. For simplicity, we assume that consumption and overtime are perfect substitutes, such that preferences depend only on *net consumption* $C = c - e$, and the agent's utility is $U(C, q) = qu(C)$, where $u(.)$ is positive, strictly increasing, and strictly concave over the domain $[-\overline{e}, \infty)$. We also assume that $\lim_{C \to -\overline{e}} u'(C) = \infty$. We normalize the attribute of the outside job to $q = 1$ and the attribute of job h to $q = \theta > 1$, which means that job h is *desirable*. Agents are born with a zero consumption endowment.

2.3 No Job Design Benchmark

Because firms are homogeneous, we consider the case of a representative firm. In each period, the firm hires a mass 1 of employees to fill vacancies in job h. Because $\underline{w}^y = \underline{w}^o = \underline{w}$, firms are indifferent between hiring young or old agents. We assume that the firm can offer either a spot contract to agents in each period or a long-term contract in which the same agents work for the firm for two consecutive periods.

Since employees are risk averse, it is optimal to offer the same wage w in each period to an employee who works in job h for both periods. Hence, the firm is indifferent between hiring the same employee for two periods or hiring a different employee (young or old) in each period. Therefore, the firm is indifferent between offering spot or long-term contracts and we thus need to characterize only the optimal spot contract.

The firm maximizes its period profit subject to the employee's participation (IR) and limited liability (LL) constraints:

$$\max_{w} R - w \tag{1}$$

subject to

$$\begin{cases} \theta u(w) \geq u(\underline{w}) & \text{IR} \\ w \geq 0 & \text{LL} \end{cases} \tag{2}$$

The optimal wage in job h is $w^a = \max\{w^*, 0\}$, where $w^* = u^{-1}\left(\frac{u(\underline{w})}{\theta}\right)$.[4] The employee "pays" for the desirable job by accepting a lower wage than her/his outside wage. This is the well-understood compensating differentials result (Rosen, 1986a). As θ increases above certain threshold ($\theta > \frac{u(\underline{w})}{u(0)}$), the limited liability constraint binds. In this case, an employee working in the desirable job benefits from rents that the employer cannot extract by further reducing the employee's wage in either period of employment.

3 Optimal Employment Contracts with Job Design

Rather than taking the production technology as a given, we can think of firms being endowed with a technology that allows them to create jobs and choose their attributes. Job design

[4] We focus only on cases where $R > w^a$ holds, that is, R is sufficiently high so that it is optimal to hire a mass one of employees in the desirable job.

is of course costly and constrained by the existing job creation technology. A firm always finds it optimal to create an *efficient job*, which is defined as a job that generates positive net surplus. The more interesting case is when a potential job (i.e., a job that could be created) is inefficient. In this case, there are no technological reasons for such a job to be created. We will show below that firms may nevertheless choose to create inefficient jobs for contractual reasons.

For simplicity, we assume here that firms have exhausted all opportunities for creating efficient jobs but can still create an unlimited number of positions in a low-quality (i.e., inefficient) job at no cost. We call this job l. The attribute of this job is set to $q_l = 1$. We also take the attribute of the desirable job as fixed.[5]

An employee who works $e \in [0, \overline{e}]$ overtime hours in job l creates revenue $y(e) < e$ for all $e \in (0, \overline{e}]$. Thus, if the firm hires a mass of N workers, each worker $i \in [0, N]$ working overtime hours e_i, then total revenue is $NE[y(e)]$, where $E[y(e)]$ is the average revenue per worker. Notice that this technology exhibits constant returns to scale (i.e., the revenue is linear in N). Constant returns to scale is a natural assumption in our setup. Increasing returns would eventually violate the assumption that the low-quality job is inefficient. Decreasing returns would imply an indirect cost of creating an additional low-quality job. Later, we explain how our results would change if the technology did not exhibit constant returns to scale. To simplify the analysis further, we restrict attention to $y(e) = ke$, with $k < 1$. All our results remain qualitatively unchanged if we assume any other functional form for $y(e)$.

Since the low-quality job is inefficient, why would firms create such jobs? Here, we show that firms may wish to create new jobs to offer workers a *career path*, which is defined as a long-term contract in which workers (potentially) switch jobs internally over time. Formally, define a long-term contract by $(x, p, w_1, w_{2l}, w_{2h}, e_{1l}, e_{2l})$, where $x \in \{l, h\}$ is the job offered in the first period of employment, $p \in [0, 1]$ is the probability of switching jobs in the second period of employment, w_1 is the wage for the first period of employment, w_{2j} is the second-period wage conditional on the employee being assigned to job $j \in \{l, h\}$ in that period, and e_{tl} is the overtime required if the employee works at job l at period $t \in \{1, 2\}$. [6] Overtime is perfectly observable and verifiable. A career path is thus a long-term contract in which

[5] See Auriol and Renault (2008) for a theory of the optimal choice of job status in an organization.

[6] Here, we use the fact that overtime is productive only in job l.

$p > 0$.

When offering long-term contracts to workers, we assume that the firm faces a no-slavery condition: A worker has the option to quit at any period. Our first result is as follows.

Lemma 1. *If the optimal contract is a career path, then it must be that $x = l$.*

This result implies that in any optimal career path, the employee is first assigned to the low-quality job and is then promoted (with some probability) to the high-quality job. The intuition for this result is as follows. The only reason for the firm to offer an inefficient low-quality job is to extract surplus that the worker enjoys from the high-quality job. When the worker is assigned to the low-quality job first, she is willing to work for less than her outside utility in the first period expecting to be promoted to the desirable job in the second period. If the ordering of jobs is reversed, then the worker would quit in the second period unless she receives a very high wage. However, the firm never finds it optimal to pay a very high wage for the low-quality job because workers' marginal utility of consumption is higher in job h than in job l.

Because all N young employees who are hired through a career path contract are assigned initially to the low-quality jobs and there is a measure one of high-quality jobs, we can interpret N as a measure of the *span of control,* which is the ratio of low-level to high-level employees.

Lemma 2. *In any optimal career path contract, $e_{1l} > 0$ and $w_1 = 0$.*

This result implies that employees will work inefficiently long hours (i.e., positive overtime) in the low-quality job in the first period of employment, while receiving the lowest possible wage compatible with limited liability.

Lemma 3. *In any optimal contract, $e_{2l} = 0$ and we can set $w_{2l} = 0$ without loss of generality.*

Lemmas 1 and 3 jointly imply that all optimal career path contracts have an up-or-out property: the employee is either "promoted" to job h in period 2 or is fired. Lemma 1 explains the "up" property: employees always start at the low-quality job. Lemma 3 explains the "out" property: employees are let go (i.e., they leave after being offered $w_{2l} = 0$) because remaining in job l is inefficient. This result contrasts with both moral hazard theories, in which "out"

is a commitment device for the firm, and selection theories, in which older employees are let go because of the option value of hiring a young agent.

Because of these three lemmas, from now on, we simplify the notation and describe a career path contract as (p,w,e), where p is the probability of being promoted to job h in period 2, w is the wage in job h in period 2, and e is the overtime in job l in period 1.

3.1 Safe Career Paths

Long-term contracts of the form $(1, w, e)$ are a special case of career paths; we call such contracts *safe career paths*. In a safe career path, the firm can commit to promoting an old employee to the desirable job if this employee supplies a certain amount of overtime while working in the firm's low-quality job.

Here, we find the conditions under which a safe career path dominates spot contracts. The firm's period program is

$$\max_{w,e} R - w + ke \tag{3}$$

subject to the following participation and limited liability constraints[7]

$$\begin{cases} u(-e) + \theta u(w) \geq 2u(\underline{w}) & IR_y \\ \theta u(w) \geq u(\underline{w}) & IR_o \\ w \geq 0 & LL \end{cases} \tag{4}$$

Proposition 1. *There exists $\overline{\theta}$ such that for all $\theta > \overline{\theta}$, a safe career path contract dominates spot contracts. The optimal safe career path is uniquely given by $(1, w^b, e^b)$ such that*

$$w^b = \max\{0, \widetilde{w}\} \quad \textit{where} \quad \tfrac{u'(-e^b)}{\theta u'(\widetilde{w})} = k \tag{5}$$

$$e^b = -u^{-1}\left[2u(\underline{w}) - \theta u(w^b)\right] > 0. \tag{6}$$

This proposition shows that for sufficiently high values of the desirable job's attribute, safe career path contracts dominate spot contracts as a mechanism for selling jobs. In an optimal safe career path, wages are increasing over time. Young agents work long hours

[7] We can ignore the upper bound for e because $\lim_{e \to \overline{e}} u'(e) = \infty$.

at the start of their careers and receive low pay in a low-quality job. Older employees are promoted to better-quality jobs, earn higher wages, and do not work overtime.

The safe career path may dominate the spot contract for two reasons. First, for a given wage for the desirable job, the safe path allows the firm to extract more of the surplus by forcing young agents to work overtime. This effect would be present even if there were no complementarity between consumption and the job attribute.[8] Second, the career path allows the employee to transfer utility across periods; consumption is relatively more valuable when an employee is old because of the complementarity with the desirable job attribute, so the employee is willing to work longer hours when young in exchange for more money when old. These two effects are essentially the conventional income and substitution effects. Both income and substitution effects are present simultaneously when the limited liability constraint does not bind, which is a case that always occurs for some parameters.

The income effect arises because an increase in θ makes the agent feel wealthier, which allows the firm to extract more surplus through more overtime. The substitution effect arises because the firm can "buy" one more unit of overtime by paying a "price" in wages in period 2. If the solution is interior, we have $\frac{u'(-e^b)}{\theta u'(w^b)} = k$ (see (5)), which implies that the firm chooses w and e such that the ratio of marginal utilities as young and old, which is the "price" of overtime hours from the firm's point of view, equals k, which is the marginal benefit of e. As θ increases, the marginal utility from wages increases in period 2, so with all else constant, for the same extra unit of overtime, the firm now faces a lower price. This makes the firm want to buy more overtime through increasing second-period wages (ignoring the income effect).

3.2 Risky Career Paths

We now consider (potentially) risky career paths. Recall that firms can create as many low-quality jobs as they wish. If a firm creates a mass N of such jobs and fills these vacancies with young agents, each employee who works in such a job faces the same probability of being promoted to the desirable job in the next period, provided the employee supplies e overtime hours. The firm offers each young employee a career path $(p = \frac{1}{N}, w, e)$.

[8] When the limited liability constraint is binding in the spot contract case, the employee captures rents that the employer cannot extract by lowering wages. Thus, a career path contract allows the employer to extract more surplus in the form of overtime.

The firm's period program is

$$\max_{w,e,p} R - w + \frac{ke}{p} \tag{7}$$

subject to the following constraints

$$\begin{cases} u(-e) + p\theta u(w) + (1-p)u(\underline{w}) \geq 2u(\underline{w}) & IR_y \\ \theta u(w) \geq u(\underline{w}) & IR_o \\ w \geq 0 & LL \\ p \in (0,1] \end{cases} \tag{8}$$

Proposition 2. *There exists a threshold $\overline{\overline{\theta}}$ such that for $\theta > \overline{\overline{\theta}}$, the optimal career path contract is uniquely given by (p^c, w^c, e^c) such that*

$$p^c = \frac{u(\underline{w}) - u(-e^c)}{\theta u(w^c) - u(\underline{w})} < 1 \tag{9}$$

$$w^c = \max\{0, \tilde{w}\}, \text{where } \frac{u'(-e^c)}{\theta u'(\tilde{w})} = k \tag{10}$$

$$e^c u'(-e^c) = u(\underline{w}) - u(-e^c) \tag{11}$$

The career path contract described in Proposition 2 exhibits the characteristics of a contest. We define the prize from winning the contest as $\eta \equiv \theta u(w^c)$. An interesting property of the risky career path equilibrium is that overtime is independent of the prize from winning the contest. To understand why this occurs, note first that an increase in η is both an income shock and a price shock. If η increases, the worker feels wealthier and is in principle willing to accept either an increase in overtime or a decrease in the probability of winning. The optimal choice of e could thus change after a change in η because of the usual income and substitution effects. Consider the income effect first. An employee's expected utility as a function of p and e is quasi-linear in e, which implies that pure income shocks do not have an impact on the optimal choice of e. Since there is no income effect on e, we need to consider only the potential substitution effect. Notice, however, that η linearly affects the marginal rate of substitution between p and e. In an optimal solution, this rate must be equal to the marginal rate of technological substitution between p and e, which is also linear in η (because

of the assumption of constant returns to scale). Changes in η have no effect on the ratio between these two rates, implying that the substitution effect is exactly zero.[9]

Corollary 1. *When risky career paths are optimal, more desirable jobs (i.e., jobs with higher θ) are associated with higher wages and wider spans of control.*

As θ increases, the marginal rate of substitution between consumption in periods 2 and 1 decreases, making consumption in period 2 relatively more valuable at the margin, and the firm thus chooses a higher wage in period 2. As both θ and w increase, the prize upon promotion increases and the firm can thus offer a lower probability of promotion while still satisfying the employees' participation constraints.

It is natural to think of high-wage, wide-span-of-control jobs as more-desirable jobs. High compensation and wide span of control may be reasons for a job to be perceived as more desirable. Our model provides a different explanation: firms choose to offer higher wages and a larger number of subordinates to people assigned to a more desirable job.

When career path contracts are offered, there is excessive employment of young agents in the sector with desirable jobs in the sense that too many young agents will work long hours in a socially inefficient task. Thus, sectors with desirable jobs are inefficiently large from a social welfare point of view. Corollary 1 shows that this inefficiency is more pronounced in sectors with more desirable job attributes.

Corollary 2. *When risky career paths are optimal, more efficient low-quality jobs (i.e., low-quality jobs with higher k) are associated with higher wages upon promotion and wider spans of control.*

An increase in k increases the marginal benefit of overtime hours, which makes the firm willing to buy more overtime by offering higher wages in period 2. As before, a higher wage upon promotion implies that the firm can offer employees a lower probability of promotion.

Corollary 3. *When risky career paths are optimal, the overtime hours worked by a young employee are independent of the probability of promotion and of the wage earned upon promotion.*

[9] If the technology exhibits decreasing returns to scale, then e increases with η. Decreasing returns to scale imply a technological cost to increasing N, which forces some of the adjustment to occur through e.

This corollary illustrates some of the differences between our theory and tournament theory (e.g., Lazear and Rosen, 1981). According to tournament theory, effort is typically not contractible and principals choose tournament prizes and winning probabilities for incentive compatibility reasons. Thus, changes in prizes and winning probabilities typically affect effort provision. In our model, because overtime is contractible, the firm chooses prizes and winning probabilities only for participation reasons. As discussed above, unless there are exogenous technological reasons for overtime to depend on the span of control (e.g., decreasing returns to scale), overtime should be largely invariant to winning probabilities and prizes.

Corollary 4. *When risky career paths are optimal, an increase in the outside wage increases overtime hours per employee and the probability of promotion and decreases the wage earned upon promotion.*

To understand the intuition, note that the cost to the firm (measured in employees' utility) of hiring N workers is proportional to $Nu(\underline{w})$, and thus, the cost of an additional employee is proportional to $u(\underline{w})$. An increase in $\underline{w}$ increases the marginal cost (or the perceived "price") of employees, so the firm chooses to hire fewer employees, that is, the probability of promotion increases. The marginal cost of overtime per employee, $u'(-e^c)$, is not affected by a change in $\underline{w}$, and thus, if $\underline{w}$ increases the firm substitutes away from N towards more overtime, e. As overtime increases in $\underline{w}$, the marginal rate of substitution between consumption in periods 2 and 1 increases, making consumption in period 2 relatively less valuable at the margin, and thus, the firm chooses a lower wage in period 2.

3.3 Optimal Contracts

We now consider the general case in which firms can offer any long-term contract they wish (spot contracts included).

Proposition 3. *The optimal contract is uniquely given by*

i) For $\theta \leq \overline{\theta}$, the firm offers a spot contract with wage w^a.

ii) For $\overline{\theta} < \theta \leq \overline{\overline{\theta}}$, the firm offers a safe career path contract $(1, w^b, e^b)$.

iii) For $\theta > \overline{\overline{\theta}}$, the firm offers a risky career path contract (p^c, w^c, e^c).

The literature on compensating differentials focuses typically on spot contracts. In that case, the optimal way to sell a desirable job is to offer employees lower wages. Proposition 3 shows that this way of selling jobs is indeed optimal in our setup for low values of θ. As a job becomes more desirable, the proposition shows that the spot contract is no longer optimal. Indeed, when the firm has the option to create low-quality jobs, career path contracts will arise endogenously as the optimal mechanism for selling jobs. Eventually, if jobs become very desirable, the optimal contract has an up-or-out feature: the employee is first assigned to a low-quality job and then promoted (with some probability) to a high-quality job. Unlike the traditional case of spot contracts, in an optimal career path contract, more desirable jobs pay higher wages.

4 Optimal Contracts when Firms Compete with Each Other

In Section 3.2, we implicitly assumed that the supply of young employees was sufficiently high such that $FN^c \leq E$, where $N^c = \frac{1}{p^c}$. We now first consider the case in which firms are constrained by the supply of young employees when choosing the optimal career path contract. Then, we also analyze the case in which competition in the product market also affects the number of firms in the sector.

4.1 Competition in the Labor Market

Because we now assume that $FN^c > E$, young workers are in short supply and all of them will find jobs in sector F, which implies that the employee's outside utility U is endogenously determined in equilibrium. Because firms are small, each firm takes U as a given when offering a contract (p, w, e) to young employees. The firm's program is

$$\max_{w,e,p} R - w + \frac{ke}{p} \tag{12}$$

subject to [10]

[10] Recall that IR_y implies IR_o.

$$\begin{cases} u(-e) + p\theta u(w) + (1-p)u(\underline{w}) \geq U & IR_y \\ w \geq 0 & LL \end{cases} \tag{13}$$

Proposition 4. *There exists a threshold $\overline{\overline{\overline{\theta}}}$ such that for $\theta > \overline{\overline{\overline{\theta}}}$, a unique symmetric equilibrium exists in which all firms offer the same contract (p^d, w^d, e^d) to young employees, where*

$$p^d = \frac{F}{E} \tag{14}$$

$$w^d = \max\{0, \tilde{w}\}, \quad \textit{where} \quad \tfrac{u'(-e^d)}{\theta u'(\tilde{w})} = k \tag{15}$$

$$e^d u'(-e^d) = p^d[\theta u(w^d) - u(\underline{w})]. \tag{16}$$

Unlike the equilibrium described in Proposition 2, here, the probability of promotion is determined by the supply of young workers and the number of firms in the sector.

Corollary 5. *An increase in the relative supply of employees ($\frac{E}{F}$) decreases overtime hours per employee and the probability of promotion and increases the wage earned upon promotion.*

An increase in the relative supply of employees is analogous to a reduction in the employees' outside option. From Corollary 4, we know that in the unconstrained case, a decrease in the outside wage decreases overtime hours and increases the wage upon promotion. The same forces are at work here.

A perhaps unexpected result is that less competition among firms increases the wages paid to those who obtain a desirable job. An increase in the supply of workers allows firms to hire more employees, who compete for promotions. Thus, from an employee's perspective, the increase in wages upon promotion is offset by the diminishing probability of being promoted. In equilibrium, competition for jobs makes employees worse off.

It is also interesting to consider the effect of competition on both inter-generational and intra-generational inequality among workers. Since all young workers are paid the same, only old workers may exhibit intra-generation inequality. Define $\theta u(w) - u(\underline{w})$ as a measure of inequality (in utility) between winners and losers among old workers. Then, less competition among firms unambiguously increases inequality because promoted employees are paid higher wages. Now, define $p\theta u(w) + (1-p)u(\underline{w}) - u(-e)$ as a measure of inter-generational inequality.

Competition among firms then increases inter-generational inequality for two reasons: both the expected prize $p\theta u(w)$ and overtime hours increase with competition.

4.2 Competition in Labor and Product Markets

In this subsection, we consider the effects of competition in the product market on labor market outcomes.[11] We endogenize the mass of firms that choose to enter sector F. Specifically, we assume that entry in the sector is costly and revenues from selling goods in the product market are a function of the mass of firms active in the sector. This assumption allows us to discuss how the cost of entry and the sensitivity of revenues to the number of active firms affect competition for young employees in the labor market and, consequently, wages in the desirable job.

Let ρ be a parameter that measures product market competition among incumbent firms. For example, ρ can be a measure of the degree of substitutability between differentiated goods or an indicator for the mode of competition (e.g., price versus quantity setting). Let $R(F, \rho)$ be the revenue associated with the high-quality job; we assume $\frac{\partial R(F,\rho)}{\partial F} < 0$ and $\frac{\partial R(F,\rho)}{\partial \rho} < 0$.

Suppose that at the beginning of the game ($t = 0$), firms that wish to operate in this sector need to pay a once-and-for-all entry cost ι, after which they can start operating in $t = 1$ in perpetuity. Let $F(\rho, \iota)$ be the mass of firms that enter the sector, given parameters ρ and ι. If firms have a discount rate of r, then the mass $F(\rho, \iota)$ is given by the zero profit condition:

$$R(F(\rho, \iota), \rho) = w - \frac{ke}{p} + r\iota. \tag{17}$$

If $F(\rho, \iota) < p^c E$, firms are unconstrained in their labor market choices, and thus, marginal changes in the parameters of competition in the product market have no effect on the optimal career path contracts. As θ increases, more firms enter the market, and each incumbent firm wants to hire more employees. Thus, for sufficiently high θ, firms will eventually become constrained, i.e., $F(\rho, \iota) > p^c E$. [12]

[11] For papers on the impact of product market competition on employment contracts and compensation, see, for example, Schmidt (1997) and Raith (2003).

[12] Formally, this result follows from the fact that p^c decreases with θ and that $-w^c + \frac{ke^c}{p^c}$ increases with θ, which implies that $F(\rho, \iota)$, as given by $R(F(\rho, \iota), \rho) = w^c - \frac{ke^c}{p^c} + r\iota$ increases with θ.

Proposition 5. *If $\theta > \overline{\overline{\overline{\theta}}}$, the wage upon promotion*

i) increases with the cost of entering the sector, ι;

ii) increases with the degree of product market competition among incumbents, ρ.

The competitiveness of an industry has two dimensions: the rivalry among incumbents and the threat of entry. Parameters ρ and ι capture these two dimensions, respectively. Proposition 5 shows that these different notions of competition have opposing effects on the wage upon promotion: tougher competition among incumbents increases the wage in the desirable job, while lower entry costs decreases the wage in the desirable job.

5 Concluding Remarks

We develop a theory of optimal employment contracts and job design. Our model is a low-friction benchmark: information is symmetric, workers are homogeneous, there is no learning of types or skill acquisition, there are no moral hazard or incentive issues, there is no exogenous uncertainty, and there are only minimal contractual restrictions. Despite its simplicity, our model provides a framework for thinking about a number of phenomena, such as internal career paths, up-or-out contracts, heavy workloads, sectorial overemployment, span of control, inequality within and between generations, and the effect of competition on employment and wages.

Our analysis can be extended in a number of ways. Introducing worker heterogeneity is one such way. For example, suppose that some workers are more skilled than others, but workers' types are discovered on the job (Jovanovic, 1979). In the optimal contract, firms commit to promoting only a subset of high-skill workers once types are revealed. If talent is not discovered on the job and is instead public information, then the equilibrium may exhibit segregation: a set of firms hires only high-skill workers, and the remaining ones hire only low-skill workers.

Another potential extension is to endogenize the choice of job attributes. For example, suppose that firms can choose how desirable a job is, subject to some cost of "desirability." This extension could have some interesting implications when firms compete for workers. A decrease in the supply of workers has two potential effects on the optimal choice of job attributes. First, because firms can now extract less surplus from employees, the marginal

benefit of increasing job desirability falls. Second, because competition for workers is now fiercer, increasing job desirability becomes more important as a means to attract workers. Thus, the total effect of competition in the labor market on firms' choice of job attributes is ambiguous.

6 APPENDIX

6.1 Proof of Lemma 1

Consider an optimal contract in which $x = h$ and an employee switches to job l with probability $p > 0$. Consider first the case in which the firm does not want to retain the employee in job h in the second period, but wants to retain the employee who switches jobs. That is, the firm offers $w_{2h} < w^a$ and w_{2l} such that the second period participation constraint is satisfied:

$$u(w_{2l} - e_{2l}) \geq u(\underline{w}). \tag{18}$$

Because $k < 1$, the firm is always better off decreasing e_{2l} and w_{2l} by the same amount, which implies that the optimal overtime is $e_{2l} = 0$.

In the first period, the worker's participation constraint must be satisfied:

$$\theta u(w_1) + pu(w_{2l}) + (1-p)u(\underline{w}) \geq 2u(\underline{w}). \tag{19}$$

A necessary condition for optimality is that $w_1 < w^a$. If $w^a = 0$, this condition is violated. If $w^a > 0$, $w_1 < w^a$ implies that (19) only holds if $w_{2l} > \underline{w}$, which implies that (18) is slack. Thus we can decrease the second period wage by $\epsilon > 0$ and increase the first period wage by the same amount, which increases the first period expected utility:

$$[\theta u'(w_1) - pu'(w_{2l})]\epsilon > 0.$$

This is strictly positive because $w_1 < w_{2l}$ and $\theta > 1 > p$. This then implies that (18) cannot be slack, contradicting the claim.

We then only need to consider the case in which $x = h$ and the firm wants to retain a

worker who does not switch jobs in the second period. If the firm wanted to retain such a worker, then it must offer $w_{2h} = w^a$. In the first period, the worker's participation constraint is

$$\theta u(w_1) + pu(\underline{w}) + (1-p)\theta u(w^a) \geq 2u(\underline{w}). \tag{20}$$

If $w^a = 0$, condition $w_1 < w^a$ is violated. If $w^a > 0$, then $\theta u(w^a) = u(\underline{w})$, thus we can simplify the participation constraint to $\theta u(w_1) \geq u(\underline{w})$, which again contradicts $w_1 < w^a$.

6.2 Proof of Lemma 2

If $e_{1l} = 0$, then the firm is always better off without a career path. Thus, it must be that $e_{1l} > 0$. Notice that if both w_1 and e_{1l} were positive, then the firm could always do better by reducing both by the same amountϵ: the agent's utility remains the same, while profit increases by $(1-k)\,\epsilon$. It follows that $w_1 = 0$.

6.3 Proof of Lemma 3

If $w_{2l} - e_{2l} \geq \underline{w}$, then an old employee assigned to job l in period 2 is retained by the firm, and thus the optimal contract must imply $e_{2l} = 0$ (see the proof of Lemma 1). Thus the firm makes an ex post loss when this employee is retained. The only reason for doing so would be to meet the employee's participation constraint

$$u(-e_{1l}) + p\theta u(w_{2h}) + (1-p)u(w_{2l}) \geq 2u(\underline{w}).$$

If we decrease w_{2l} by a small amount and decrease first-period overtime by the same small amount, the gain in expected utility is

$$u'(-e_{1l}) - (1-p)u'(w_{2l}) > 0,$$

which then implies that the firm is always better off by reducing w_{2l} if the worker is retained, which implies that the worker is never retained in job l in the second period. Thus, without loss of generality, we may set $w_{2l} = 0$.

6.4 Proof of Proposition 1

The firm's period program is:

$$\max_{w,e} \pi = R - w + ke \tag{21}$$

subject to

$$\begin{cases} u(-e) + \theta u(w) \geq 2u(\underline{w}) & IR_y \\ \theta u(w) \geq u(\underline{w}) & IR_o \\ w \geq 0 & LL \end{cases}. \tag{22}$$

Notice that IR_y implies IR_o, and that IR_y must bind in an optimal solution. The problem then simplifies to

$$\max_{w,e,\lambda,\delta} R - w + ke + \lambda[u(-e) + \theta u(w) - 2u(\underline{w})] + \delta w. \tag{23}$$

The first-order conditions are

$$\begin{cases} -1 + \lambda\theta u'(w) + \delta = 0 \\ k - \lambda u'(-e) = 0 \\ u(-e) + \theta u(w) - 2u(\underline{w}) = 0 \\ w \geq 0 \end{cases}. \tag{24}$$

We first consider that $w > 0$ and derive the condition for an interior solution $e = e^b > 0$ to exist. Define $h \equiv u^{-1}$and from IR_y find $w = w^b$ as

$$w^b(-e^b) = h\left[\frac{2u(\underline{w}) - u(-e^b)}{\theta}\right] \tag{25}$$

$$u'(-e^b) = k\theta u'\left(h\left[\frac{2u(\underline{w}) - u(-e^b)}{\theta}\right]\right). \tag{26}$$

One can verify that the second-order conditions hold and that, if an interior solution for e exists ($e > 0, w \geq 0$), it is unique. Since the left-hand side of (26) is strictly increasing in e^b, with $\lim_{e\to\overline{e}} u'(-e) = \infty$, and the right-hand side is strictly decreasing, a necessary and

sufficient condition for an unique interior solution to exist is

$$u'(0) < k\theta u'(w^b(0)). \tag{27}$$

Define $\theta' \equiv u'(0)/ku'(w^b(0))$. For $\theta > \theta'$, then a unique interior solution for e always exists when $w > 0$.

Define

$$\theta^* = \frac{2u(\underline{w}) - u(0)}{u(0)}, \tag{28}$$

and consider the following cases:

Case 1: $\theta^* \leq \frac{1}{k}$. If $\theta < \theta^*$, $\theta u(0) < 2u(\underline{w}) - u(0)$, that is, IR_y is not satisfied at $w = 0$, thus we need w to be strictly positive. Because $\theta < \theta'$ (recall that $\theta' > \frac{1}{k}$) implies that we cannot have both w and e positive at the same time, then we have $e = 0$. Under a spot contract, profit is $R - w^a$. Because $w^a \leq w^b$, and since $e = 0$, then $\pi^b < \pi^a$.

If $\theta = \theta^*$, then the contract $w = e = 0$ is feasible and we have that $w^a = 0$ (because $\theta^* > \frac{u(\underline{w})}{u(0)}$), thus $\pi^b \geq \pi^a$.

If $\theta > \theta^*$, $\theta u(0) > 2u(\underline{w}) - u(0)$, which means that at $e = 0$ the agent enjoys rents, which can be extracted by increasing e. Thus, π^b is strictly increasing in θ because the firm always has the option to offer $w = 0$ and overtime implied by $u(-e^b) = 2u(\underline{w}) - \theta u(0)$, which is strictly increasing in θ, which implies $\pi^b > \pi^a$ if and only if $\theta > \theta^*$. Thus, in Case 1, we have $\overline{\theta} = \theta^*$.

Case 2: $\theta^* > \frac{1}{k}$. We first prove that $\theta' \leq \theta^*$. Suppose that $\theta' > \theta^*$, and that $\theta \in (\theta^*, \theta')$. This implies that the optimal contract must be such that $e^b = 0$ and $w^b(0) > 0$. But, because $\theta > \theta^*$, then some $e > 0$ and $w = 0$ are feasible, which contradicts the assumption that e^b and w^b are optimal. Thus, $\theta' \leq \theta^*$.

If$\theta \leq \theta'$, then $e^b = 0$ and, because $\theta' \leq \theta^*$, we have $w^b(0) > 0$. Since $w^b(0) > w^a$, then $\pi^b < \pi^a$. This means that the firm should choose to offer a spot contract.

If $\theta > \theta'$, the profit difference between the safe career path and the spot contract is

$$\pi^b - \pi^a = -\max\left\{h\left[\frac{2u(\underline{w}) - u(-e^b)}{\theta}\right], 0\right\} + ke^b + \max\left\{h\left[\frac{u(\underline{w})}{\theta}\right], 0\right\}, \tag{29}$$

which (from the envelope theorem) is strictly increasing in θ when $w^b > 0$. When $w^b = 0$

(and since $w^b \geq w^a$), $\pi^b - \pi^a = ke^b$, which is strictly increasing in θ (from (26)). Finally, $lim_{\theta \to \infty} \pi^b - \pi^a = k\overline{e} > 0$. So in Case 2, $\overline{\theta}$ is implicitly defined by making (29) equal to zero. We note that $\overline{\theta} > \theta'$.

6.5 Proof of Proposition 2

The firm's period program is:

$$\max_{w,e,p} \pi = R - w + \frac{ke}{p} \tag{30}$$

subject to

$$\begin{cases} u(-e) + p\theta u(w) + (1-p)u(\underline{w}) \geq 2u(\underline{w}) & IR_y \\ \theta u(w) \geq u(\underline{w}) & IR_o \\ w \geq 0 & LL \\ p \in [0,1] \end{cases} . \tag{31}$$

First we notice that as before IR_y implies IR_o. Here we solve for the cases where $p < 1$; the conditions for this to be the case will be derived below. The firm solves:

$$\max_{w,e,p,\lambda,\delta} R - w + \frac{ke}{p} - \lambda(u(\underline{w}) - u(-e) - p(\theta u(w) - u(\underline{w}))) + \delta w, \tag{32}$$

which simplifies as follows:

$$\begin{cases} -1 + \lambda p \theta u'(w) + \delta = 0 \\ \frac{k}{p} - \lambda u'(-e) = 0 \\ -\frac{ke}{p^2} + \lambda(\theta u(w) - u(\underline{w})) = 0 \\ p = \frac{u(\underline{w}) - u(-e)}{\theta u(w) - u(\underline{w})} \end{cases} \tag{33}$$

To find the optimal overtime e^c, we replace $\lambda = \frac{k}{pu'(-e)}$ and $p = \frac{u(\underline{w})-u(-e)}{\theta u(w)-u(w)}$ into $-\frac{ke}{p^2} + \lambda(\theta u(w) - u(\underline{w})) = 0$ and simplify:

$$e^c u'(-e^c) + u(-e^c) = u(\underline{w}). \tag{34}$$

From (34) we notice that e^c is independent of θ, k and w. The left-hand side of the equation is increasing in overtime. Indeed, let $A(e) \equiv eu'(-e) + u(-e)$, $\frac{\partial A}{\partial e} = -eu''(-e) > 0$. For $e = 0$, $A(0) = u(0)$ and $u(0) < u(\underline{w})$. For $e = \overline{e}$, $A \to \infty$. It follows that a unique positive solution for e exists.

The unique optimal choice for the second period wage is $w^c = \max\{0, \tilde{w}\}$, where

$$\theta k u'(\tilde{w}) = u'(-e^c). \tag{35}$$

The right-hand side of equation (35) is independent of $\tilde{w}$, while the left-hand side is decreasing in $\tilde{w}$. We note that w^c is (non-strictly) increasing in θ and k:

$$\frac{\partial \tilde{w}}{\partial \theta} = -\frac{u'(\tilde{w})}{u''(\tilde{w})\theta} > 0, \tag{36}$$

$$\frac{\partial \tilde{w}}{\partial k} = -\frac{u'(\tilde{w})}{u''(\tilde{w})k} > 0. \tag{37}$$

Note that since $\tilde{w}$ is strictly increasing in θ, it follows that the LL constraint only binds if $\theta \leq \frac{u'(-e^c)}{ku'(0)}$. We note that $\frac{u'(-e^c)}{ku'(0)} > \frac{1}{k}$.

The optimal promotion probability is:

$$p^c = \frac{u(\underline{w}) - u(-e^c)}{\theta u(w^c) - u(\underline{w})}. \tag{38}$$

We see that p^c is strictly decreasing in θ. Therefore there exists a threshold $\overline{\overline{\theta}}$, for which $p^c = 1$, such that for $\theta > \hat{\theta}$, p^c is strictly lower than 1. The threshold is implicitly defined by:

$$\overline{\overline{\theta}} = \frac{2u(\underline{w}) - u(-e^c)}{u(w^c)}. \tag{39}$$

6.6 Proof of Corollary 1 and 2

When risky career paths are optimal, the optimal wage and probability of promotion are given by:

$$w^c = \max\{0, \tilde{w}\}, \quad \text{where} \quad \frac{u'(-e^c)}{\theta u'(\tilde{w})} = k \tag{40}$$

$$p^c = \frac{u(\underline{w}) - u(-e^c)}{\theta u(w^c) - u(\underline{w})} \tag{41}$$

Since e^c is not affected by changes in θ, it follows that:

$$\frac{\partial \tilde{w}}{\partial \theta} = -\frac{u'(\tilde{w})}{\theta u''(\tilde{w})} > 0 \Rightarrow \frac{\partial w^c}{\partial \theta} \geq 0 \tag{42}$$

This result and the fact that $p^c = \frac{u(\underline{w})-u(-e^c)}{\theta u(w^c)-u(\underline{w})}$ implies that $\frac{\partial p}{\partial \theta} < 0$ and, therefore, $\frac{\partial N}{\partial \theta} > 0$.

Since e^c is not affected by changes in k, it follows that:

$$\frac{\partial \tilde{w}}{\partial k} = -\frac{u'(\tilde{w})}{k u''(\tilde{w})} > 0 \Rightarrow \frac{\partial w^c}{\partial k} \geq 0 \tag{43}$$

This result and the fact that $p^c = \frac{u(\underline{w})-u(-e^c)}{\theta u(w^c)-u(\underline{w})}$ implies that $\frac{\partial p}{\partial k} \leq 0$ and, therefore, $\frac{\partial N}{\partial k} \geq 0$.

6.7 Proof of Corollary 3 and 4

When risky career paths are optimal, the optimal overtime is given by

$$e^c u'(-e^c) = u(\underline{w}) - u(-e^c), \tag{44}$$

thus the optimal overtime is independent of w^c and p^c.

From equation 44 we find that:

$$\frac{\partial e^c}{\partial \underline{w}} = \frac{u'(\underline{w})}{e^c \frac{\partial u'(-e^c)}{\partial e^c}} > 0. \tag{45}$$

Using this result and $\frac{\theta u'(\tilde{w})}{u'(-e^c)} = \frac{1}{k}$, we find that:

$$\frac{\partial \tilde{w}}{\partial \underline{w}} = \frac{\frac{\partial u'(-e^c)}{\partial e^c}}{\theta k u''(\tilde{w})} \frac{\partial e^c}{\partial \underline{w}} < 0 \Rightarrow \frac{\partial w^c}{\partial \underline{w}} \leq 0. \tag{46}$$

Since $\frac{\partial e^c}{\partial \underline{w}} > 0$, $\frac{\partial w^c}{\partial \underline{w}} \leq 0$, and $p^c = \frac{u(\underline{w})-u(-e^c)}{\theta u(w)-u(\underline{w})}$, it follows that $\frac{\partial p^c}{\partial \underline{w}} > 0$.

6.8 Proof of Proposition 3

From Proposition 1, we know that if $\theta \leq \overline{\theta}$, then the spot contract dominates the safe career path contract. From Proposition 2, we know that for $\theta > \overline{\overline{\theta}}$ the risky career path contract dominates the safe career contract. Thus, it is sufficient to show that $\overline{\overline{\theta}} > \overline{\theta}$.

Case 1: $\theta^* = \frac{2u(\underline{w})-u(0)}{u(0)} < \frac{1}{k}$: From the proof of Proposition 1, we know that $\overline{\theta} = \theta^*$, which implies $\overline{\theta} < \frac{1}{k}$. If $\overline{\overline{\theta}} \leq \frac{u'(-e^c)}{ku'(0)}$, $w^c = 0$ and from (39) we have $\overline{\overline{\theta}} = \frac{2u(\underline{w})-u(-e^c)}{u(0)}$. Thus $\overline{\overline{\theta}} = \frac{2u(\underline{w})-u(-e^c)}{u(0)} > \frac{2u(\underline{w})-u(0)}{u(0)} = \overline{\theta}$. If, instead, $\overline{\overline{\theta}} > \frac{u'(-e^c)}{ku'(0)}$, then $\overline{\overline{\theta}} > \frac{1}{k} > \overline{\theta}$.

Case 2: $\theta^* = \frac{2u(\underline{w})-u(0)}{u(0)} \geqslant \frac{1}{k}$: If we have $\frac{2u(\underline{w})-u(-e^c)}{u(0)} \leq \frac{u'(-e^c)}{ku'(0)}$, then the solution at $\theta = \overline{\overline{\theta}} = \frac{2u(\underline{w})-u(-e^c)}{u(0)}$ is such that $(w^c = 0, e^c > 0)$, which implies $-w^c + ke^c = ke^c > 0$. Since $w^a \geq 0$, it follows that $\pi^c = R - w^c + ke^c > R - w^a = \pi^a$. From Proposition 1 we know that $\overline{\theta}$ is implicitly defined by $\pi^b = \pi^a$, thus $\overline{\overline{\theta}} > \overline{\theta}$.

If instead $\frac{2u(\underline{w})-u(-e^c)}{u(0)} > \frac{u'(-e^c)}{ku'(0)}$, then the solution at any $\theta > \overline{\overline{\theta}}$ must be $(w^c > 0, e^c > 0)$, which implies that $\overline{\overline{\theta}} = \frac{2u(\underline{w})-u(-e^c)}{u(w^c)}$ is such that $\overline{\overline{\theta}} \in \left(\frac{u'(-e^c)}{ku'(0)}, \frac{2u(\underline{w})-u(-e^c)}{u(0)}\right)$ (because $\frac{2u(\underline{w})-u(-e^c)}{u(w^c)}$ attains its maximum at $w^c = 0$). Define $\tilde{e}$, such that:

$$\frac{2u(\underline{w}) - u(-\tilde{e})}{u(0)} = \frac{u'(-\tilde{e})}{ku'(0)}.$$

It can be verified that $\tilde{e}$ exists and is unique. To show that, at $\theta = \frac{2u(\underline{w})-u(-\tilde{e})}{u(0)} = \frac{u'(-\tilde{e})}{ku'(0)}$, $\tilde{e} < e^c$, suppose it is not, then $(w = 0, e^c > 0)$ is also feasible and strictly better than $(w^c > 0, e^c > 0)$, which contradicts the assumption that the latter is an optimal solution. This implies that $\frac{u'(-\tilde{e})}{ku'(0)} < \frac{u'(-e^c)}{ku'(0)} < \overline{\overline{\theta}}$. Because at $\theta = \frac{u'(-\tilde{e})}{ku'(0)}$, we know that $\pi^b > \pi^a$ (because overtime is positive and wages are zero), then we know that $\overline{\theta} < \frac{u'(-\tilde{e})}{ku'(0)}$. Thus $\overline{\overline{\theta}} > \overline{\theta}$.

6.9 Proof of Proposition 4

From the proof of Proposition 2, we know that p^c decreases with θ. Therefore, there exists $\theta = \overline{\overline{\overline{\theta}}}$, given by $\frac{F}{E} = p^c$ and such that for $\theta > \overline{\overline{\overline{\theta}}}$ we have $\frac{F}{E} > p^c$, which means that young workers are in short supply and thus firms cannot choose p^c in equilibrium. This implies the equilibrium must be such that $p^d = \frac{F}{E}$.

An equilibrium in this economy is a vector (U, p^d, w^d, e^d) such that (i) given U, (p^d, w^d, e^d)

are chosen to maximize (12) subject to (13), (ii) $p^d = \frac{F}{E}$, and

$$\text{(iii) } U = \max\left\{u(-e^d) + p^d\theta u(w) + (1-p^d)u(\underline{w}), 2u(\underline{w})\right\}.$$

We express p from the participation constraint as

$$p = \frac{U - u(\underline{w}) - u(-e)}{\theta u(w) - u(\underline{w})}, \tag{47}$$

thus we can rewrite the principal's period program as

$$\max_{e,w} R - w + ke\frac{\theta u(w) - u(\underline{w})}{U - u(\underline{w}) - u(-e)}.$$

The first order conditions with respect to w and e are as follows:

$$\frac{ke\theta u'(w)}{U - u(\underline{w}) - u(-e)} = 1, \tag{48}$$

$$\frac{eu'(-e)}{U - u(\underline{w}) - u(-e)} = 1. \tag{49}$$

It follows that, for a given U, the optimal values for (p, w, e) are given by:

$$\begin{cases} w = \max\{0, \tilde{w}\} & \theta k u'(\tilde{w}) = u'(-e) \\ eu'(-e) = U - u(\underline{w}) - u(-e) & \\ p = \frac{U - u(\underline{w}) - u(-e)}{\theta u(w) - u(\underline{w})} & \end{cases}. \tag{50}$$

The analysis regarding the existence and uniqueness of e and w is similar to the analysis in the unconstrained case and therefore omitted. We differentiate $eu'(-e) = U - u(\underline{w}) - u(-e)$ with respect to U and find that (after some algebra):

$$\frac{\partial e}{\partial U} = -\frac{1}{eu''(-e)} > 0. \tag{51}$$

For the case where $w = \tilde{w}$, we have that

$$\frac{\partial w}{\partial U} = -\frac{u''(-e)\frac{\partial e}{\partial U}}{\theta k u''(w)} < 0. \tag{52}$$

In equilibrium, it must be that

$$U = \max\left\{u(-e(U)) + u(\underline{w}) + \frac{F}{E}(\theta u(w(U)) - u(\underline{w})), 2u(\underline{w})\right\}.$$

Since the unconstrained optimal probability p^c is lower than $\frac{F}{E}$, it follows that

$$U = u(-e(U)) + u(\underline{w}) + \frac{F}{E}(\theta u(w(U)) - u(\underline{w})) > 2u(\underline{w}).$$

From (51) and (52) it follows that $u(-e(U)) + u(\underline{w}) + \frac{F}{E}(\theta u(w(U)) - u(\underline{w}))$ is decreasing in U. Hence, a solution to $U = u(-e(U)) + u(\underline{w}) + \frac{F}{E}(\theta u(w(U)) - u(\underline{w}))$ exists.

6.10 Proof of Corollary 5

From $p^d = \frac{F}{E}$, it follows that p^d decreases with $\frac{E}{F}$. We have shown already that $\frac{\partial w}{\partial U} < 0$ and $\frac{\partial e}{\partial U} > 0$. Since U increases with $p^d = \frac{F}{E}$, it follows that $\frac{\partial w}{\partial \frac{E}{F}} > 0$ and $\frac{\partial e}{\partial \frac{E}{F}} < 0$.

6.11 Proof of Proposition 5

(i) From Corollary 5, we know that in the case in which young workers are in short supply, $\frac{\partial w}{\partial p} < 0$. From (17) and $\frac{\partial R(F,\rho)}{\partial F} < 0$ we see that if the entry cost ι increases, the mass of firms entering the sector $F(\rho, \iota)$ decreases. A lower $F(\rho, \iota)$ implies a lower p and therefore a higher wage in the desirable job.

(ii) Since $\frac{\partial R(F,\rho)}{\partial \rho} < 0$, an increase in ρreduces $F(\rho, \iota)$; a lower $F(\rho, \iota)$ implies a lower p and therefore a higher wage in the desirable job.

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