The theory of exploitation as the unequal exchange of labour

Roberto Veneziani and Naoki Yoshihara

School of Economics and Finance

Queen Mary
University of London

The theory of exploitation as the unequal exchange of labour*

Roberto Veneziani[†]   Naoki Yoshihara[‡]

May 15, 2017

Abstract

This paper analyses the normative and positive foundations of the theory of exploitation as the unequal exchange of labour (UEL). The key intuitions behind all of the main approaches to UEL exploitation are explicitly analysed as a series of formal claims in a general economic environment. It is then argued that these intuitions can be captured by one fundamental axiom - called *Labour Exploitation* - which defines the basic domain of all UEL exploitation forms and identifies the formal and theoretical framework for the analysis of the appropriate definition of exploitation.

JEL classification: D63; B51.

Keywords: Exploitation, Unequal Exchange of Labour, axiomatic analysis.

[†]School of Economics and Finance, Queen Mary University of London, Mile End Road, London E1 4NS, UK. E-mail: r.veneziani@qmul.ac.uk

[‡](Corresponding author) Department of Economics, University of Massachusetts Amherst, 200 Hicks Way, Amherst, MA 01003, USA; The Institute of Economic Research, Hitotsubashi University, Kunitachi, Tokyo 186-0004 Japan; and School of Management, Kochi University of Technology, Kochi 782-8502, Japan. E-mail: n_yoshihara_1967@yahoo.co.jp

1 Introduction

What is exploitation? In political philosophy, the most general definition affirms that A exploits B if and only if A takes unfair advantage of B. Despite its intuitive appeal, this definition leaves two major issues in need of a precise specification, namely the kind of unfairness involved and the structure of the relationship between A and B that allows A to take advantage of B. There is considerable debate in the economic and philosophical literature on both issues. Although both aspects of exploitative relations are arguably crucial, the analytical focus of this paper is on the unfairness, or more precisely, on the economic inequalities involved in the concept of exploitation.[1]

To be specific, this paper analyses the concept of *exploitation as an unequal exchange of labour* (hereafter, UEL). At the most general level, according to UEL exploitation theory, exploitative relations are characterised by systematic differences between the amount of labour that individuals 'give' to the economy, in some relevant sense, and the amount of labour that they 'receive', in some relevant sense. Yet, there are many conceivable ways of defining the amounts of labour given and received by agents, and many alternative approaches have indeed been proposed. In his seminal book on exploitation theory alone, Roemer [32] examines no fewer than six distinct UEL definitions,[2] and many other definitions have been proposed in the formal literature on exploitation theory.[3]

Given the nature of many debates in exploitation theory, alternative UEL definitions may seem to differ for relatively minor, and merely technical details (for example, concerning the appropriate notion of labour content in general economies). At a closer look, however, some deep theoretical cleavages emerge and different UEL approaches incorporate such distinct

---

[1]Alternatively, one may interpret our analysis as focusing on the *measurement* of exploitation, which is relevant even in approaches that reject the idea that exploitation is (primarily) an injustice (e.g., Holmstrom [18]; Reiman [29]; Wood [50], fn.15). For a discussion of the structural aspects of exploitation, see Arneson [1]; Warren [49]; Veneziani [45]. Domination is central in the approach developed by Vrousalis [48].

[2]In addition to the standard definition that applies to subsistence Leontief economies ([32], Part I), see Roemer ([32], pp.121, 132-133, 168).

[3]The literature is too vast for a comprehensive list of references. Classic contributions include Morishima [25]; Duménil [6]; Foley [15]; Roemer [31]; and Flaschel [11, 12]. For a discussion, see Yoshihara [52, 53].

normative and positive intuitions that it is legitimate to wonder whether they actually bear any family resemblance. In some approaches, exploitation is defined as a *property* of individuals and "refers to the relationship between a person and society as a whole as measured by the transfer of the person's labor to the society, and the reverse transfer of society's labor to the person" (Roemer [34], p.31). In others, exploitation is primarily a *relation* between individuals (e.g., Holmstrom [18]; Fleurbaey [14]). The essential normative content of the notion of UEL exploitation is also contested. According to Elster ([8], p.167), "Being exploited means, fundamentally, working more hours than are needed to produce the goods one consumes," and thus exploitative relations are affected by saving/investment decisions. Other authors, instead, emphasise purchasing power, rather than consumption, and the idea that "workers give more labor to their employers than they receive through the goods their wages can afford" (Fleurbaey [14], p.653). In more traditional, Marxian approaches, the emphasis is on unpaid, or surplus labour, defined as the labour in excess of the amount necessary to produce the means of subsistence for workers and their families, and investment decisions do not affect exploitation (e.g., Holmstrom [18]). And so on.

This paper explores the foundations of UEL exploitation in an abstract economic environment. The main purpose is to identify the core of UEL exploitation theory that is shared by *all* of the main approaches in the literature. The (often implicit) intuitions incorporated in the various definitions are rigorously formalised as separate *Claims*, and thoroughly examined. Then, an axiom called *Labour Exploitation* is formulated, which summarises the foundations of UEL exploitation theory (the basic *Claims*). The axiom sets weak restrictions on the way in which the set of exploiters and the set of exploited agents are identified in economies with heterogeneous agents and a general production technology. It is interpreted as a minimal necessary condition - a *domain* condition - to capture the core intuitions of exploitation theory and it is shown that indeed all of the main approaches satisfy it.

It is important to stress at the outset that, although we believe that there are good reasons to focus on labour as the variable of normative interest in economic interactions, and in many economic contexts the notion of exploitation is inextricably linked with some form of labour exchange,[4] our aim is *not* to defend the normative relevance of the distributive

---

[4]Tellingly, after criticising the UEL approach and suggesting to replace it with a definition based on property relations (PR), in later contributions, Roemer has acknowledged the importance of labour in exploitation theory. He has proposed that "an agent is exploited in the Marxist sense, or capitalistically exploited, *if and*

aspects of UEL exploitation theory. Nor does the paper propose a specific UEL definition. Rather, our main objective is to clarify a number of views and intuitions that are widely, albeit implicitly held in the literature, and to provide a rigorous formal and conceptual framework for the analysis of UEL exploitation. Although this leaves the questions of the injustice of the allocation of labour and of the appropriate UEL definition open, a clear statement of the foundations of UEL theory is necessary in order to tackle both issues.[5]

This paper represents a first important step in the development of a novel axiomatic framework to analyse some key issues in exploitation theory.[6] For, if *Labour Exploitation* provides a domain condition capturing the fundamental intuitions of UEL theory common to all approaches, then the appropriate definition can be characterised by imposing additional desirable properties (axiomatic restrictions) within the class of admissible exploitation forms identified by *Labour Exploitation*. We return to this issue in sections 6 and 7 below.

2 The structure of UEL approaches

In order to illustrate the key conceptual issues, consider the baseline case of a two-class economy in which one good, say corn, is produced by means of itself and labour. To be specific, a units of corn seed and L units of (a single type of homogeneous) labour are necessary to produce one unit of corn, where $a < 1$. Assume that all agents have the same skills. Suppose further that capitalists save and accumulate, while all workers earn a subsistence wage and consume the same (subsistence) amount of corn c. Under these rather stringent assumptions, the definition of UEL exploitation is relatively uncontroversial. Let v denote the amount of labour necessary to produce one unit of corn: this comprises both the amount of labour L directly necessary in production and the amount of labour contained in

_____

only if PR holds and the exploiter gains by virtue of the labor of the exploited" (Roemer [35], p.96). For, "the expenditure of effort is characteristically associated with exploitation" (Roemer [35], fn.11).

[5]Moreover, a clear analysis of the foundations of the notion of exploitation may "help understand social conflicts, because the protesters often voice complaints motivated by the feeling of being at the wrong end of some form of exploitation" (Fleurbaey [14], p.673).

[6]An axiomatic approach was long overdue in exploitation theory, where new definitions have sometimes emerged as the result of a somewhat ad hoc process of adjustment of the theory to various problems and counterexamples (such as Steedman's [42] famous proof that a negative aggregate surplus value could occur alongside positive aggregate profits). Exceptions include Hollander [17] and, more recently, Yoshihara and Veneziani [54]; Veneziani and Yoshihara [47]; and Yoshihara [52].

the corn seed, va. Formally, $v = va + L$ and so $v = L(1-a)^{-1}$. Then agent ν is exploited (resp., an exploiter) if and only if the labour she expends, λ^ν, is greater (resp., lower) than the labour contained in c, vc.[7]

The definition of an agent's exploitation status requires the identification of *three* separate normative benchmarks. The first is the amount of labour that the agent gives, or contributes - which, in this setting, is simply equated to the labour time performed in economic activities, λ^ν. The definition of the amount of labour received, instead, requires the definition *both* of a relevant bundle (or, more generally, bundles) received by the agent, which can be called the *exploitation reference bundle(s)* (henceforth, ERB), *and* of the labour associated with, or contained in it - here, respectively, c and vc.

In the simple economy considered, all three benchmarks are relatively uncontroversial.[8] As soon as the restrictive assumptions on technology, behaviour and endowments are relaxed, however, the appropriate definition of the labour 'given' and 'received' by agents is not obvious, and several approaches have indeed been proposed. For example, if agents have heterogeneous skills, or different types of labour inputs are used in production, then perhaps the labour given by agents should be the *effective* (e.g. skill-adjusted) labour that they have contributed, rather than the *time* they expended in production. But maybe not.

Similarly, if more general technologies are considered (allowing, for example, for joint products), the naïve generalisation of the standard approach to the definition of labour received can yield paradoxical results - such as bundles containing a negative amount of labour, as famously shown by Steedman [42] - and so various definitions of the labour contained in a *given* bundle have been proposed, focusing either on actual production activities or on some feasible, possibly counterfactual, technology. As Roemer ([33], p.283) puts it, "If the production set is complicated in various ways, then it is not possible to give an unambiguous definition of what 'labor-embodied' means".

Finally, if agents do not consume a given, equal subsistence bundle, then the choice of

---

[7]In this paper, we follow the literature and consider exploitative relations within a given time period. For a detailed analysis of the complications arising in a dynamic setting, see Elster [8] and Veneziani [44, 45].

[8]This holds also in n-good economies with a $n \times 1$ subsistence bundle c and a standard Leontief technology (A, L), where A is a square $n \times n$ nonnegative, productive matrix and L is a strictly positive $1 \times n$ vector describing, respectively, the amount of each input and the labour necessary to produce one unit of each of the n goods. In this case the amount of labour directly and indirectly contained in each good is $v = L(I - A)^{-1}$, where I is the identity matrix.

the ERB itself is not obvious: one may focus either on agents' actual choices or on some alternative (affordable) bundle, or indeed bundles. The former approach takes a subjectivist view by emphasising agents' actual choices in the determination of their exploitation status. Proponents of the latter approach object that a subjectivist perspective makes exploitation depend on idiosyncratic *consumption* decisions so that agents who consume different bundles but are otherwise identical may end up having a different exploitation status.

In the next sections, we clarify and reconstruct the foundations of UEL exploitation theory in an abstract economic environment. We state them as formal claims that incorporate logically weak restrictions on the definition of the three normative benchmarks - the labour 'given' by agents, the ERB(s), and the labour associated with the ERB(s) - that are shared by all of the main approaches. Our purpose is not to discriminate among competing views, but rather to define the theoretical boundaries of UEL theory. Formally, we aim to identify a general *domain* for all admissible UEL exploitation forms.

3   Unequal exchange of labour: foundations

We aim to analyse UEL exploitation theory at an abstract level. Hence, in what follows, we impose only minimal restrictions on endowments, technology and preferences, and make no assumptions concerning agents' behaviour - agents may be standard utility maximisers, or they may be characterised by some form of bounded rationality. Nor do we restrict the analysis exclusively to private market economies, and agents may adopt a variety of institutional arrangements to coordinate economic activity.

Consider an abstract economic environment. Let $\mathcal{N} = \{1, ..., N\}$ be the set of *agents*, with generic element $\nu \in \mathcal{N}$, and let $\mathcal{M} = \{1, ..., M\}$ be the set of produced *commodities* in the economy. Let $\mathbb{R}$ denote the real numbers. Let P be the production set, describing the technologically feasible production techniques. Elements of P - activities - are vectors denoted as α, which describe the amount of *effective* labour α_l and the vector of produced goods $\underline{\alpha}$ used as inputs in order to produce a vector of outputs $\overline{\alpha}$.[9] We follow the literature and assume that technology displays constant returns to scale.[10] Denote the vector of net

---

[9]Formally, $\alpha = (-\alpha_l, -\underline{\alpha}, \overline{\alpha})$ where $\alpha_l \in \mathbb{R}_+$ is the effective labour input; $\underline{\alpha} \in \mathbb{R}_+^M$ are the inputs of the produced goods; and $\overline{\alpha} \in \mathbb{R}_+^M$ are the outputs of the M goods.

[10]Formally, P is a closed, convex cone with $\mathbf{0} = (0, ..., 0) \in P$.

outputs (outputs minus inputs) arising from α as $\widehat{\alpha} \equiv \overline{\alpha} - \underline{\alpha}$.

Let $\omega \in \mathbb{R}_+^M$ be a semi-positive vector of social endowments of productive assets. Let $l^\nu > 0$ describe the labour endowment of each agent $\nu \in \mathcal{N}$.[11] More precisely, for all $\nu \in \mathcal{N}$, let $s^\nu > 0$ be agent ν's skill level. If $\overline{L} > 0$ is the amount of time each agent is endowed with, then $l^\nu = s^\nu \overline{L}$. Similarly, if λ^ν is the amount of *time* expended by ν in production, then $\Lambda^\nu = s^\nu \lambda^\nu$ is the amount of *effective* labour performed by ν.

Each economy has a list $\langle \mathcal{N}, \mathcal{M}, P, \omega, (l^\nu)_{\nu \in \mathcal{N}} \rangle$. Given this list, the set of *feasible economic allocations* can be identified and is denoted by $\mathcal{Z} \subseteq \mathbb{R}_+^{NM+N}$: a vector $\boldsymbol{z} \equiv (\boldsymbol{x}, \boldsymbol{\Lambda}) \in \mathcal{Z}$ represents an allocation $\boldsymbol{x} = (x^\nu)_{\nu \in \mathcal{N}}$ of the M commodities to the N agents in the economy and a profile $\boldsymbol{\Lambda} = (\Lambda^\nu)_{\nu \in \mathcal{N}}$ of (effective) labour supplied by each agent $\nu \in \mathcal{N}$ such that there exists a production activity $\alpha \in P$ satisfying $\underline{\alpha} \leqq \omega$, $\widehat{\alpha} = \sum_{\nu \in \mathcal{N}} x^\nu$, $\alpha_l = \sum_{\nu \in \mathcal{N}} \Lambda^\nu$, and $0 \leqq \Lambda^\nu \leqq l^\nu$ for each agent $\nu \in \mathcal{N}$.[12] In other words, an economic allocation is feasible if the aggregate amount of each commodity allocated to agents can be produced given the aggregate endowments of physical and human capital in the economy.

The first claim of UEL theory requires that a definition of exploitation unambiguously identifies the exploitation status of every agent at any feasible economic allocation.

Claim 1. (*The exploitation partition*) At any allocation $\boldsymbol{z} \in \mathcal{Z}$, any definition of exploitation identifies a *partition* of the set of agents. Formally, there exist three pairwise disjoint subsets of $\mathcal{N}$, denoted as $\mathcal{N}^{ter}, \mathcal{N}^{ted}, \mathcal{N}^n$ such that $\mathcal{N}^{ter} \cup \mathcal{N}^{ted} \cup \mathcal{N}^n = \mathcal{N}$ and, for every agent $\nu \in \mathcal{N}$, either $\nu \in \mathcal{N}^{ter}$ and ν is an exploiter; or $\nu \in \mathcal{N}^{ted}$ and ν is exploited; or $\nu \in \mathcal{N}^n$ and ν is exploitation-neutral.

According to Claim 1, even though exploitation may be a property of aggregates, such as social classes, such aggregates are ultimately made of individuals with a well-defined exploitation status. Claim 1 is thus weakly individualistic in that it excludes radically holistic (Wright *et al* [51]) approaches, which identify the existence of exploitation *only* at the aggregate level, without any reference to (coalitions of) individuals.[13] Claim 1 does

---

[11] Although we allow for different labour *skills*, we are implicitly ruling out different *types* of labour used in production. This is only for simplicity and all of the key insights of the paper can be extended to economies with different labour inputs. For a thorough discussion see Veneziani and Yoshihara [46].

[12] The notation for vector inequalities is: for all $y, y' \in \mathbb{R}^p$, $y \geqq y'$ if and only if $y_i \geqq y_i'$ for all i; $y \geq y'$ if and only if $y \geqq y'$ and $y \neq y'$; $y > y'$ if and only if $y_i > y_i'$ for all i.

[13] Some applications of UEL theory focus on exploitative relations between *countries* (e.g. Roemer [32],

not rule out approaches viewing "exploitation as a social phenomenon, and the existence of exploitation need not imply, in principle, the existence of individual exploiters or exploited" (Roemer [32], p.136). But it does require the exploitation status of each agent to be clearly defined: each agent must belong to one of three exploitation categories and no agent can belong to more than one category.[14]

Claim 1 defines exploitation as a *property*, not primarily as a *relation* (Elster [8], p.174). It requires exploitation status to be defined at an overall allocation, and is silent about exploitation in individual transactions (unlike in approaches focusing on just acquisition, such as Steiner [43], or disadvantage, such as Goodin [16] and Sample [38]). This does not exclude the possibility that exploitation be diagnosed relationally, at the level of individual transactions, and agents may be exploited in some transactions, and exploiters in others. According to Claim 1, however, the agents' overall exploitation status should also be identified taking into account all of their economic activities.[15]

In principle, Claim 1 may apply to any definitions of exploitation, including those that focus on fair pricing (such as Joan Robinson's [30] classic approach and, more recently, Reiff [28]) or property relations (e.g. Roemer [32], chapter 7). The next claim restricts the field of analysis as it defines the theory of exploitation as the unequal exchange of *labour*.

Claim 2. (*UEL exploitation*) At any allocation $z \in \mathcal{Z}$, the exploitation status of an agent $\nu \in \mathcal{N}$ is identified by comparing the labour that ν 'gives', $L_G^\nu(z)$, and the labour that ν 'receives', $L_R^\nu(z)$.

Claim 2 incorporates an "idea of exploitation, as a certain kind of lack of reciprocity" (Cohen [3], p.343), whereby an agent is exploited whenever "the labor he contributes in one form does not return to him in another form" (Husami [19], p.44). The lack of reciprocity may

chapter 1). This is consistent with Claim 1 provided countries are seen as collections of agents.

[14]Claim 1 rules out the possibility of incommensurable dimensions, or 'spheres', of exploitation: agents can be exploiters in one dimension and exploited in a different dimension, but their overall exploitation status must be well-defined and unique.

[15]It may be tempting to argue that because the overall exploitation status logically depends on the outcome of many transactions, then exploitation status *must* also be defined at the level of individual transactions. This is a nonsequitur: the fact that, from a positive viewpoint, the overall exploitation status depends on the outcome of many transactions does not imply that exploitation status can, or should be defined at the level of individual transactions in a normatively relevant way. Yet, as noted above, Claim 1 is consistent with approaches that identify exploitation also in individual transactions. See, e.g., Fleurbaey ([14], p.661).

8

be normatively relevant per se,[16] or because differences in labour 'given' and 'received' reflect normatively significant inequalities. Claim 2 holds in approaches that define exploitation as the (forced) extraction of surplus, or unpaid labour (Holmstrom [18]; Elster [9]; Reiman [29]), but it is more general than that and it does not imply a commitment to an objective theory of value. For the amounts of labour 'given' and 'received' by agents need not be determined objectively, and may reflect agents' preferences and beliefs. Indeed, Claim 2 does not imply a commitment to *any* theory of value, or to the claim that labour is the only thing that produces value.[17] It only identifies labour as the key normative variable of interest - the main unit of account of exploitation theory - and thus rules out approaches based on wealth, income, utility, and so on. In exploitation theory, labour accounting is the "way of characterizing what it is that people give one another ... (where 'give' is understood very broadly to refer to any way in which some person undergoes a loss that ends up a gain to another)" (Reiman [29], p.9).[18]

Claim 2 requires any UEL approach to determine two variables for each agent: the amount of labour given *and* the amount received. Neither choice is obvious - especially if one allows for heterogeneous skills and general technologies - and different views can be proposed. Let the vectors $L_G(z) = (L_G^1, ..., L_G^N)$ and $L_R(z) = (L_R^1, ..., L_R^N)$ denote, respectively, the labour 'given' and the labour 'received' by each agent at allocation $z \in \mathcal{Z}$.[19] In all UEL approaches, the labour 'given' by agent $\nu \in \mathcal{N}$ is just a scalar. In simple economies with homogeneous

---

[16] "Exploitation is wrong because ... [t]he capitalist gets something for nothing, or much for little, at the expense of others" (Elster [9], p.39). More generally, UEL exploitation may reflect a deviation from some fair reference norm in exchange (Fleurbaey [14], p.666). According to Arneson ([1], p.213), "Exploitation is one important form of mistreatment. To be exploited is roughly to be forced to perform drudgery to an unfairly great extent, and to receive in return an unfairly small share of goods" (as measured in the labour numéraire).

[17] See, for example, Buchanan ([2], p.46); Roemer ([34], p.62); Cohen ([3], ch.11); Fleurbaey ([13], p.181; [14], p.654); Vrousalis ([48], p.138).

[18] This is what Amartya Sen ([40], p.177) has called the "descriptive interpretation of the labour theory of value," whose main purpose is to provide, on the one hand, "a description of production that focuse[s] on human beings" and, on the other hand, the foundations of a normative approach whereby "personal participation can be seen as the basis of entitlement of fruits of production." For an alternative justification of labour accounting in exploitation theory, see Roemer ([32], pp.283-288).

[19] Whenever this entails no ambiguity, for each $\nu \in \mathcal{N}$, we write L_G^ν and L_R^ν instead of $L_G^\nu(z)$ and $L_R^\nu(z)$ for the sake of notational simplicity.

labour, it is equal to the labour *time* expended by ν in production.[20] In more general settings, however, various definitions of $L_G(\boldsymbol{z})$ can be adopted, which incorporate different normative intuitions. In order to focus on the core intuitions of UEL exploitation theory shared by all approaches, we leave $L_G(\boldsymbol{z})$ unspecified while assuming that L_G^ν is a scalar for all $\nu \in \mathcal{N}$. We return to this issue in section 6 below.

The definition of the labour that each agent 'receives' is more controversial, and many different proposals can be (and have been) made *even in economies with homogeneous labour*. For the labour 'received' by an agent at an allocation $\boldsymbol{z} \in \mathcal{Z}$ is not necessarily captured by a single number and each L_R^ν could be a (possibly infinite) set of labour amounts related, for example, to an agent's set of affordable bundles. There is therefore significant scope for major theoretical differences in the definition of $L_R(\boldsymbol{z})$. However, some theoretically relevant restrictions can be identified that are common to all of the main approaches.

In all UEL approaches, for agent ν to be classified as exploited, she must 'give' more labour - and thus have a higher L_G^ν - than it would be necessary to classify her as an exploiter, *everything else equal* (that is, for given consumption, income, preferences, and so on). Formally, *exploitation status* is said to *be increasing in $L_G(\boldsymbol{z})$* if for any two allocations $\boldsymbol{z}, \boldsymbol{z}' \in \mathcal{Z}$ such that $\boldsymbol{x} = \boldsymbol{x}'$, $L_R(\boldsymbol{z}) = L_R(\boldsymbol{z}')$, and $\sum_{\nu \in \mathcal{N}} \Lambda^\nu = \sum_{\nu \in \mathcal{N}} \Lambda'^\nu$, the following three statements hold: for each agent $\nu \in \mathcal{N}$, (i) $L_G^\nu(\boldsymbol{z}) > L_G^\nu(\boldsymbol{z}')$ holds whenever ν is exploited at $\boldsymbol{z}$ and is an exploiter at $\boldsymbol{z}'$; (ii) ν is exploited at $\boldsymbol{z}$ whenever $L_G^\nu(\boldsymbol{z}) > L_G^\nu(\boldsymbol{z}')$ and ν is exploited at $\boldsymbol{z}'$; and (iii) ν is an exploiter at $\boldsymbol{z}'$ whenever $L_G^\nu(\boldsymbol{z}) > L_G^\nu(\boldsymbol{z}')$ and ν is an exploiter at $\boldsymbol{z}$. Then:

Claim 3. (*Labour monotonicity*) Exploitation status is increasing in $L_G(\boldsymbol{z})$.

Claim 3 is considered to be so obvious that it is never explicitly stated in the literature. That it embodies a core intuition of UEL theory is particularly evident in Roemer's ([32], part I) subsistence economies in which all agents receive the same amount of labour in all

---

[20]Some UEL approaches - such as Emmanuel's [10] classic analysis of international inequalities - focus exclusively on the sphere of exchange and consider as exploitative any transactions in which bundles of goods are exchanged which embody different amounts of labour. In these approaches, $L_G(\boldsymbol{z})$ and $L_R(\boldsymbol{z})$ correspond to the labour embodied in the bundles that agents give up and receive in exchange, respectively. Although the axiomatic framework proposed in this paper encompasses these approaches, too, we shall not analyse them in detail because they focus mostly on international relations and, more importantly, their normative foundations are unclear (see Roemer [32] and Schweickart [39]).

equilibrium allocations and perform different amounts of labour depending on their physical endowments. Or in his sequence of examples with subsistence consumption but different institutional arrangements and distributions of productive assets (e.g., Roemer [33]). Yet, Claim 3 is clearly shared by all UEL approaches.[21]

Together with Claim 1, *Labour monotonicity* implies that, at any allocation $z \in \mathcal{Z}$, the exploitation status of each agent $\nu \in \mathcal{N}$ can be defined by identifying *at most* two cut-off values $L^{\nu}_{\min}(z), L^{\nu}_{\max}(z)$ such that ν is exploited if and only if $L^{\nu}_G(z) > L^{\nu}_{\max}(z)$ while ν is an exploiter if and only if $L^{\nu}_G(z) < L^{\nu}_{\min}(z)$. In other words, the information contained in the potentially complex object $L^{\nu}_R(z)$ can be reduced to a focus on the upper and lower bounds of the labour 'received' by ν, $L^{\nu}_{\min}(z), L^{\nu}_{\max}(z)$, which can be called the *reference labour amounts*.[22]

By Claims 1 and 3, we can reformulate the core idea of UEL exploitation.[23]

Claim 2′. (*UEL exploitation*) At any allocation $z \in \mathcal{Z}$, for all $\nu \in \mathcal{N}$, there exist $L^{\nu}_{\min}, L^{\nu}_{\max}$ with $L^{\nu}_{\min} \leqq L^{\nu}_{\max}$ such that agent ν is an exploiter if and only if $L^{\nu}_G < L^{\nu}_{\min}$; exploited if and only if $L^{\nu}_G > L^{\nu}_{\max}$; and exploitation-neutral if and only if $L^{\nu}_{\min} \leqq L^{\nu}_G \leqq L^{\nu}_{\max}$.

Claim 2′ provides a precise statement of UEL exploitation theory at a general level. However, in order to have an operational definition, it is necessary to impose further theoretical restrictions on $L_R(z)$. This is by no means trivial. For there are up to $2N$ reference labour amounts potentially 'received' by agents at any allocation $z \in \mathcal{Z}$. So the question is; how are $(L^{\nu}_{\min}(z), L^{\nu}_{\max}(z))_{\nu \in \mathcal{N}}$ determined? Or, equivalently, how do agents 'receive' labour?

Let $\mathcal{C} \subseteq \mathbb{R}^M$ be the set of conceivable bundles of commodities, and let $C(z) = (C^1, ..., C^N)$ describe the *set of consumption opportunities* for each agent at $z \in \mathcal{Z}$, where $C^{\nu} \subseteq \mathcal{C}$, for all $\nu \in \mathcal{N}$.[24] For each agent $\nu \in \mathcal{N}$, C^{ν} may describe either the actual allocation of commodities to ν, or more generally a set of bundles that capture ν's consumption possibilities at x.[25] The next claim states that at an allocation $z \in \mathcal{Z}$ the amount of labour received by each

---

[21]Formally, while all other claims considered in this paper hold at a given allocation, Claim 3 embodies a property of UEL exploitation *across allocations*. This is standard in the literature where the properties of exploitative relations are often analysed by comparing different (possibly counterfactual) allocations.

[22]That is, for any $z \in \mathcal{Z}$, either $L^{\nu}_R(z) \in [L^{\nu}_{\min}(z), L^{\nu}_{\max}(z)]$ or $L^{\nu}_R(z) \subseteq [L^{\nu}_{\min}(z), L^{\nu}_{\max}(z)]$ holds.

[23]For every $\nu \in \mathcal{N}$, $L^{\nu}_{\min} \leqq L^{\nu}_{\max}$ follows from the disjointness of $\mathcal{N}^{ter}, \mathcal{N}^{ted}, \mathcal{N}^n$ in Claim 1.

[24]Again, we are writing C^{ν} instead of $C^{\nu}(z)$ for notational simplicity.

[25]Strictly speaking, C^{ν} captures "allocation" opportunities, for the M goods in the economy are not necessarily consumption goods. With this proviso, we use the more descriptive term "consumption opportunities."

agent depends on her consumption opportunities.

Claim 4. (*Sensitivity to consumption opportunities*) At any allocation $z \in \mathcal{Z}$, there exist a $C(z)$ and a profile of correspondences $F = (F^1, ..., F^N)$ such that $L_R^\nu = F^\nu(C^\nu)$, for all $\nu \in \mathcal{N}$.

Claim 4 states that agents 'receive' labour via (sets of) commodity bundles that capture - in some relevant sense yet to be specified - their consumption possibilities, and it requires that the labour received by each agent, L_R^ν, be related to a (set of) *reference commodity bundle(s)*, C^ν. "[E]xploitation theory views goods as vessels of labor, and calculates labor accounts for people by comparing the 'live' labor they expend in production with the 'dead' labor they get back in the vessels" (Roemer [34], p.31).

In some Marxist approaches, for example, the labour received by workers - necessary labour - is determined by looking at "the value of those goods required for [their] own subsistence" (Buchanan [2], p.37). Hence C^ν is a singleton and coincides with "the subsistence basket taking into account historical as well as moral considerations" (Desai [5], p.21). Claim 4 is more general: C^ν could be a *set* of vectors, and it could be related to the agents' subjective preferences, choices, and income, or it could be just a conceivable (set of) reference bundle(s).[26] Thus, Claim 4 also captures the intuitions of approaches that do not explicitly focus on the labour embodied in specific bundles of goods, such as the 'New Interpretation' (Duménil [6]; Foley [15]), according to which L_R^ν is equivalent to the amount of social labour agents receive a claim to via their income - that is, their income multiplied by the value of money.

Claim 4 says nothing about the relation between C^ν and L_R^ν. For each agent ν and each C^ν, F^ν is a purely normative construct that identifies the labour 'received' by ν based on the set of consumption opportunities C^ν. Claim 4 does not specify how F^ν translates the sets of consumption opportunities into amounts of labour: F^ν could be either a (single-valued) function or a (set-valued) correspondence; no restrictions are imposed on its shape; and it is allowed to be agent-specific. Given Claims 2' and 3, however, we know that, for each agent, at most two reference amounts of labour are necessary in order to define her exploitation

---

[26]It is worth stressing again that Claims 1-4 (and indeed Claims 5-9 below) do not imply a committment to an objectivist approach to exploitation, as they allow subjective tastes, choices, and beliefs to determine the key elements of a UEL account. More on this in section 6 below.

status. Then, by Claim 4 it is reasonable to require that all relevant information in C^ν be summarised into at most two reference bundles, for each agent ν.

Claim 5. (*Exploitation reference bundles, ERBs*) At any allocation $z \in \mathcal{Z}$, for each $\nu \in \mathcal{N}$, there exist $c^\nu_{\min}, c^\nu_{\max} \in C^\nu$ and a function f^ν such that $L^\nu_{\min} = f^\nu(c^\nu_{\min})$ and $L^\nu_{\max} = f^\nu(c^\nu_{\max})$.

Claim 5 implies that, for each agent $\nu \in \mathcal{N}$, the reference labour amounts, $L^\nu_{\min}, L^\nu_{\max}$, can be interpreted as the amounts of labour associated with, or contained in some theoretically relevant reference commodity bundle(s), the ERBs. In the standard view, an input-output approach is adopted to determine employment multipliers which uniquely identify the *unit* labour content of each commodity, and a fortiori the labour content of *any* bundles of goods (for a discussion, see Morishima [24, 25]; Flaschel [11, 12]). Claim 5 is much weaker: it only requires that any definition of UEL exploitation has a well-defined (albeit possibly implicit) notion of labour associated with, or contained in, the ERBs.

Claims 1 through to 5 represent the basic conceptual framework of UEL exploitation theory. In UEL theory, every agent is characterised by the tuple, $< L^\nu_G, L^\nu_R >$, which determines her exploitation status (Claims 1 and 2). Different approaches can be interpreted as different ways of specifying such tuple. However, in general, *ceteris paribus*, if an agent is exploited, she gives more labour than if she is an exploiter (Claim 3). For any $\nu \in \mathcal{N}$, the labour received by ν, L^ν_R, is related to ν's consumption opportunities (Claim 4). Further, although L^ν_R could be a large (potentially infinite) set, in order to determine exploitation status it is sufficient to focus on at most two values, $L^\nu_{\min}, L^\nu_{\max}$, for each agent (Claim 2$'$). Thus, for each $\nu \in \mathcal{N}$, the amount of labour received is determined by the amount of labour associated with, or contained in some relevant commodity bundle(s), the ERBs (Claim 5).[27]

4   The exploitation reference bundles

Claims 1 through to 5 are not a complete theory of exploitation. They provide a rigorous framework to conceptualise UEL exploitation, and the theoretical choices of different approaches, in terms of restrictions on $< L^\nu_G, L^\nu_R >$. Concerning L^ν_R, Claim 5 implies that in order to define the exploitation status of each agent, it is necessary both to select the relevant ERBs, $c^\nu_{\min}, c^\nu_{\max}$, *and* to identify their labour content - that is, the (possibly implicit)

---

[27]Recall that, for the time being, no restriction is imposed on $L_G(z)$.

function f^ν that associates a reference labour amount with each ERB. In general economies, neither choice is obvious, and various definitions have, in fact, been proposed. In this section, we explore some additional restrictions on c^ν_{min}, c^ν_{max} and f^ν that aim to incorporate formally weak, theoretically robust, and widely shared intuitions in UEL exploitation theory.

For each agent ν, let $\omega^\nu \in \mathbb{R}^M_+$ be ν's endowment of productive assets. Let (p, w, r) be the vector describing, respectively, the prices of commodities and labour, and the rate of return on capital at allocation $\boldsymbol{z} \in \mathcal{Z}$.[28] Then, let $B(\omega^\nu, l^\nu; p, w, r)$ be ν's set of *potentially affordable* bundles or, in market economies, her potential budget set at $\boldsymbol{z}$.[29] The next claim requires c^ν_{min}, c^ν_{max} to be in the set of economically feasible choices for agent ν.

Claim 6. (*Economic feasibility*) At any allocation $\boldsymbol{z} \in \mathcal{Z}$ with a price vector (p, w, r), $c^\nu_{min}, c^\nu_{max} \in B(\omega^\nu, l^\nu; p, w, r)$ for all $\nu \in \mathcal{N}$.

Claim 6 stipulates that the ERBs, which determine the amount of labour 'received' by agent ν - and, ultimately, agent ν's exploitation status, - be related to the agent's affordable choices or, in market economies, to her income. Claim 6 clearly holds in UEL approaches that take the ERB to be unique and equal to the workers' subsistence basket.[30] But it is also true in more general approaches that abandon a subsistence view and define labour received by workers focusing on "commodities whose value is equivalent to the wages he receives" (Buchanan [2], p.37), or "the amounts of wage goods which they can buy with the wages they receive" (Morishima [24], p.46). As Roemer ([35], p.90) aptly puts it, at the most general level, labour received by an agent is determined looking at the amount of labour contained "in the goods he can purchase with his revenues from production (which may come from wages, profits, or the sale of commodities)".

Yet Claim 6 does not require that c^ν_{min}, c^ν_{max} exhaust income, i.e. be on the budget line, nor does it constrain the way in which they should be chosen. By focusing on *potential*, rather than actual or realised income, Claim 6 allows for many different views concerning the

---

[28] Because we are not assuming economic activities to be coordinated by competitive markets, (p, w, r) could be either actual or hypothetical (shadow) prices and we could further generalise the analysis by allowing for differential wage rates, profit rates and even prices of commodities. Yet this would significantly increase notational intensity without adding further theoretical insights.

[29] Formally: $B(\omega^\nu, l^\nu; p, w, r) \equiv \{c^\nu \in \mathcal{C} \mid pc^\nu \leqq wl^\nu + rp\omega^\nu\}$.

[30] Indeed, the weak notion of economic feasibility formalised in Claim 6 is (implicitly) endorsed even in approaches that deny the relevance of income distribution for the determination of exploitation status, and the rate of exploitation, while focusing on the concept of necessary labour (e.g., Holmstrom [18], pp.360ff).

appropriate choice of the ERBs: they may coincide with the bundle actually purchased by agents at $\boldsymbol{z} \in \mathcal{Z}$,[31] or they may measure some potential, or normatively relevant, commodity bundles that are actually or only potentially affordable.[32] The latter approach is important, for example, in *modal* definitions in which exploitation status "cannot be established just by looking at actual behaviour" (Elster [8], p.173).

Claim 6 imposes an *economic* feasibility condition on the ERBs. Following classic UEL exploitation theory, Claim 7 requires that $c_{\min}^{\nu}$, $c_{\max}^{\nu}$ be also *technologically* feasible as net output of some production process. For any $c \in \mathbb{R}_+^M$, let $\phi(c)$ be the set of activities that produce at least c as net output.[33] Then:

Claim 7. (*Technical feasibility*) At any allocation $\boldsymbol{z} \in \mathcal{Z}$, $\phi(c_{\min}^{\nu}) \neq \varnothing$ and $\phi(c_{\max}^{\nu}) \neq \varnothing$ holds for all $\nu \in \mathcal{N}$.

The emphasis on production conditions is standard in the traditional Marxist view according to which the amount of labour received by workers is equal to the value of labour power. Because labour power is a produced commodity, its value is given by the amount of labour necessary to produce it, which is "identical with the amount of time required to produce the means of subsistence of the worker" (Cohen [3], p.340; see also Morishima [25], p.614). According to Roemer ([31], p.54), "the value of labor power possesses a complex determination, whose origins are in the method of production." But, more generally, in the UEL approach exploitation is a specific wrong rooted in the sphere of production (Cohen [3], p.345; Elster [8], p.167; Fleurbaey [13], p.184), and formal UEL definitions "map from the fundamental relations of production into the labour market and from this to the market of commodities, and then invert the mapping so as to go from the market of the final goods back to the production relationship" (Morishima and Catephores [26], p.43).

Claim 7 requires the ERBs to be technically feasible as *net* output of some production

---

[31] According to Elster [8], for example, the ERBs should be related to agents' actual consumption choices. For "the use made of the revenues may be relevant to the moral status of their distribution" (Elster [8], p.177) and one may wish to distinguish between consumption and investment.

[32] Yet, Claim 6 incorporates an emphasis on current economic data that contradicts approaches focusing on *historical* injustice, such as Steiner's [43]. For Claim 6 requires that the ERBs (and thus the amount of labour received by agents) be determined based on actual, current endowments rather than, counterfactually, on the endowments that would have emerged had no prior rights violation occurred.

[33] Formally, $\phi(c) \equiv \{\alpha \in P \mid \widehat{\alpha} \geqq c\}$.

15

process.[34] This is because the direct labour used to produce a bundle c as net output allows one to capture the *total* amount of labour contained in the ERBs, namely "the embodied labour - direct and indirect - in producing c from scratch" (Roemer [32], p.148). A focus on net output is also natural in those UEL approaches that define embodied labour such that total labour performed in the economy in a given period is equal to the labour contained in net national product, and conceive of exploitation as measuring (using a labour metric) how net national product is parcelled out to individuals.[35]

Claims 6 and 7 impose restrictions on the choice of ERBs, but are silent on the labour contained in $c^\nu_{\min}$, $c^\nu_{\max}$. The next two claims focus on the function f^ν that associates a reference amount of labour to each ERB.

Let $\boldsymbol{s} = \left(s^1, ..., s^N\right)$. Claim 8 is a natural complement to Claim 7 in that it requires that the reference amounts of labour also be related to production conditions.

Claim 8. (*Reference labour amounts*) At any allocation $\boldsymbol{z} \in \mathcal{Z}$, the reference amounts of labour $L^\nu_{\min} = f^\nu(c^\nu_{\min})$ and $L^\nu_{\max} = f^\nu(c^\nu_{\max})$ depend on the amount of labour necessary to produce $c^\nu_{\min}$, $c^\nu_{\max}$. Formally, for all $\nu \in \mathcal{N}$, there exist production processes $\alpha^\nu_{\min}$, $\alpha^\nu_{\max} \in P$ such that $\alpha^\nu_{\min} \in \phi\left(c^\nu_{\min}\right)$ and $\alpha^\nu_{\max} \in \phi\left(c^\nu_{\max}\right)$, and a profile of positive linear transformations $\left(\kappa^\nu\left(P, \boldsymbol{s}, \boldsymbol{z}\right)\right)_{\nu \in \mathcal{N}}$ such that $L^\nu_{\min} = \kappa^\nu\left(P, \boldsymbol{s}, \boldsymbol{z}\right) \cdot \alpha^\nu_{l\min}$ and $L^\nu_{\max} = \kappa^\nu\left(P, \boldsymbol{s}, \boldsymbol{z}\right) \cdot \alpha^\nu_{l\max}$.

Claim 8 imposes a restriction on the function f^ν that associates a reference labour amount to every ERB by stipulating that it be constrained by the properties of the production process. The notion of labour contained adopted is by no means metaphysical and bears no conceptual relation with the disputes on the labour theory of value. The labour contained in the ERBs is a well-defined amount related to current technical conditions,[36] and to the direct amount of (effective) labour used in the production process. Contra Steedman [42], Claim 8 rules out negative reference labour amounts, because the labour contained in the ERBs, and thus the labour received by agents "should be non-negative by definition" (Morishima and Catephores [26], p.32; see also Flaschel [12], p.18).

---

[34]Claim 7 may be considered as a strong requirement if each agent has different bundles $c^\nu_{\min}$, $c^\nu_{\max}$. Yet it is easily satisfied for realistic choices of $c^\nu_{\min}$, $c^\nu_{\max}$, or if the set P possesses some standard properties (such as free disposal; see Veneziani and Yoshihara [47]).

[35]For example, Flaschel [11, 12]; Duménil [6]; Foley [15]; and Fleurbaey ([13], p.171). See also the discussion in Morishima and Catephores ([26], pp.42-43) and Roemer ([34], pp.30-31).

[36]Rather than the labour actually used in the past to produce a certain bundle of goods (Cohen [3], pp.346ff).

According to Claim 8, the reference labour amounts may be quantities of effective labour, but this is not a requirement: different specifications of the positive linear transformations $(\kappa^\nu(P, \boldsymbol{s}, \boldsymbol{z}))_{\nu \in \mathcal{N}}$ will correspond to different notions of the labour 'received' (as well as 'given') by agents, and the appropriate choice of $(\kappa^\nu(P, \boldsymbol{s}, \boldsymbol{z}))_{\nu \in \mathcal{N}}$ may depend on the normative use of the concept of exploitation (Fleurbaey [13], pp.176-177).[37] We return to this issue in section 6 below.

A focus on production and on the notion of labour *contained* in the ERBs may seem restrictive in that it rules out, for example, approaches focusing on exchange whereby the reference amounts of labour are determined based on labour *commanded* - that is, on the amount of labour power that an agent may purchase on the market with her income. This should not be a major concern. Although they do not enter Claim 8, considerations of purchasing power or, more general, affordability are relevant in the framework proposed so far, especially in Claim 6. In general, Claim 8 allows for the possibility that price magnitudes and market relations enter the definition of labour contained in the ERBs (Roemer [32]; Flaschel [11, 12]). Further, although Claim 8 does impose some restrictions on f^ν, they are extremely weak and the choice of reference amounts of labour remains significantly open. For, there may be many ways of producing the ERBs, and so the sets $\phi(c_{\min}^\nu)$ and $\phi(c_{\max}^\nu)$ may be very large.[38] Claim 8 imposes no restrictions on the way in which $L_{\min}^\nu$ and $L_{\max}^\nu$ are chosen among the (possibly infinite) labour amounts of activities in $\phi(c_{\min}^\nu)$ and $\phi(c_{\max}^\nu)$.

Given that our aim is to identify the foundations of UEL exploitation theory, and derive a domain condition that incorporates the intuitions of all of the main approaches, there is no reason to impose any strong restrictions on f^ν. Different restrictions may appropriately reflect alternative views on UEL exploitation, or may be desirable in different economic environments.[39] Yet, a common assumption in the UEL literature is that, however determined,

---

[37]Actually, the axiomatic framework could be generalised further by allowing for nonlinear transformations $(\kappa^\nu(P, \boldsymbol{s}, \boldsymbol{z}))_{\nu \in \mathcal{N}}$ in Claim 8, or by comparing L_R^ν and L_G^ν by means of a general binary relation $\succsim$ (which is not necessarily equal to the inequality $\geqq$), and letting $L_{\min}^\nu = \alpha_{l \min}^\nu$ and $L_{\max}^\nu = \alpha_{l \max}^\nu$ in Claim 8. Given the focus of this paper, however, we have opted for the more transparent approach in terms of inequalities and restricting Claim 8 to linear transformations, consistently with the rest of the literature.

[38]If the production set P satisfies a standard free disposal condition, then for any $c \in \mathbb{R}_+^M$, the set $\phi(c)$ may have an uncountably infinite number of elements.

[39]Maybe reference labour amounts should be determined based on the production techniques *actually* used at a given allocation (see, e.g., Flaschel [11, 12]). But maybe not. Claim 8 allows for approaches focusing on

the reference amounts of labour be well-defined and unique.

Claim 9. (*Production objectivism*) At any allocation $z \in \mathcal{Z}$, for any $\nu, \mu \in \mathcal{N}$, if $c^\nu_{\min} = c^\mu_{\min}$ and $c^\nu_{\max} = c^\mu_{\max}$, then $L^\nu_{\min} = L^\mu_{\min}$ and $L^\nu_{\max} = L^\mu_{\max}$.

According to Claim 9, if the ERBs of two agents are equal, then the associated reference labour amounts should be the same: the labour associated with, or contained in the ERBs should be uniquely identified based on production conditions and not on idiosyncratic subjective factors. To put it differently, Claim 9 does not impose any restrictions on how the labour contained in the ERBs should be defined, but it does require that it be uniquely determined. Although there are many conceivable ways of determining $L^\nu_{\min}$ and $L^\nu_{\max}$, Claim 9 states that such differences should be at the level of the theory, and not of arbitrary individual characteristics. It is important to stress that this does not imply that subjective preferences and individual choices are irrelevant in determining exploitation status. It only means that the labour contained in the ERBs should not depend on the identity of the agents' receiving them.

Claim 9 holds in the standard Marxian approach, whereby the labour content of a bundle, however defined, is *uniquely* determined and exploitation status is independent of agents' subjective characteristics. According to Morishima ([24], p.181), for example, a fundamental property of a definition of exploitation is that "each worker has to be shown to be equally exploited by the capitalists." Yet, Claim 9 seems reasonable also more generally and it is widely shared in UEL approaches: as argued by Yoshihara and Veneziani [55], one of the essential, if not defining characteristics of exploitation as a normative construct in political philosophy is its (weakly) objectivist thrust.[40] "If an agent could change from being exploited into being an exploiter simply as a result of a change of tastes, some of the moral connotations of exploitation would be lost" (Elster [8], p.174). Claim 9 seems the weakest, and most easily defensible, part of a general objectivist theory of UEL exploitation. For if labour is 'received' via some commodity bundles, then it seems natural to require that a single reference amount of labour be associated with any ERB.

Claims 6-9 complete the analysis of the basic structure of UEL approaches. In UEL

---

possibly counterfactual (e.g. 'optimal' in some relevant sense) techniques, as in Definitions 1 and 2 below.

[40] More strongly, Roemer ([32], p.110) has argued that the notion of exploitation "should be independent of the subjective preferences of agents, as whether an agent is exploited should be an objective fact over which he has no control."

exploitation theory, the exploitation status of an agent $\nu \in \mathcal{N}$ is characterised by the tuple, $< L_G^\nu, L_R^\nu >$, denoting the labour 'given' and 'received' by her. The labour received by each agent ν, L_R^ν, is related to ν's consumption opportunities and is determined by the amount of labour associated with, or contained in some relevant commodity bundle(s), the ERBs. Claims 6-9 provide some basic restrictions on the choice of the ERBs, *and* on their labour content. For every agent ν, the ERBs should be potentially affordable (Claim 6) and technically feasible (Claim 7). The labour associated with, or contained in the ERBs is (a linear function of) the amount of labour necessary to produce them (Claim 8), which should be well-defined and uniquely determined independently of subjective factors (Claim 9).[41]

As shown below, Claims 1 through to 9 are theoretically salient and identify the contours of UEL exploitation theory, while defining a rather large domain of admissible approaches.

5 Labour exploitation

Claims 1 through to 9 identify the common structure underlying different approaches to UEL exploitation. In this section, we formalise an axiom, called *Labour Exploitation*, that incorporates Claims 1-9. The axiom is a *domain condition*: it captures the core intuitions of UEL theory and so, we argue, it identifies the domain of admissible exploitation forms.

Labour Exploitation (LE): *Given any definition of exploitation, the set of exploited agents $\mathcal{N}^{ted} \subseteq \mathcal{N}$ and the set of exploiters $\mathcal{N}^{ter} \subseteq \mathcal{N}$ should have the following property. Given an economy with $(P, \boldsymbol{s})$, at any allocation $\boldsymbol{z} \in \mathcal{Z}$ with a price system (p, w, r), there exist a profile of positive linear transformations $(\kappa^\nu(P, \boldsymbol{s}, \boldsymbol{z}))_{\nu \in \mathcal{N}}$ and a profile $(c_{\min}^\nu, c_{\max}^\nu)_{\nu \in \mathcal{N}}$ satisfying production objectivism such that for each agent $\nu \in \mathcal{N}$, $c_{\min}^\nu, c_{\max}^\nu \in B(\omega^\nu, l^\nu; p, w, r)$ and for some $\alpha^{c_{\min}^\nu} \in \phi(c_{\min}^\nu)$ and some $\alpha^{c_{\max}^\nu} \in \phi(c_{\max}^\nu)$ with $\alpha_l^{c_{\max}^\nu} \geqq \alpha_l^{c_{\min}^\nu}$:*

$$\nu \in \mathcal{N}^{ted} \text{ if and only if } \kappa^\nu(P, \boldsymbol{s}, \boldsymbol{z}) \cdot \alpha_l^{c_{\max}^\nu} < L_G^\nu,$$
$$\nu \in \mathcal{N}^{ter} \text{ if and only if } \kappa^\nu(P, \boldsymbol{s}, \boldsymbol{z}) \cdot \alpha_l^{c_{\min}^\nu} > L_G^\nu.$$

---

[41]It should be emphasised that Claims 7-9 do *not* provide a *general* theory of the determination of labour content. For they do not focus on *all* conceivable or even feasible bundles of commodities $c \in \mathbb{R}_+^M$. Indeed, they impose restrictions only on a (potentially very small) subset, $\{c_{\min}^\nu, c_{\max}^\nu\}$, of the set C^ν of consumption opportunities of each agent ν. Moreover, they hold at a given allocation $\boldsymbol{z} \in \mathcal{Z}$, and impose no restrictions on the determination of labour content across allocations.

LE captures the core insights of UEL exploitation theory common to *all* of the main approaches.[42] It summarises Claims 1-9: at any feasible allocation $\boldsymbol{z}$, the exploitation status of every agent ν is determined by the difference between the amount of labour that ν 'gives' to the economy, and the amount she 'receives'. Whereas the former quantity is left unspecified, **LE** requires that the labour received by agents be determined by identifying two (possibly identical) ERBs, $c^\nu_{\min}$, $c^\nu_{\max}$, which must be (at least potentially) affordable and technically feasible, and their associated labour content which is equal to (a linear transformation of) the labour necessary to produce them as net output, $\kappa^\nu(P, \boldsymbol{s}, \boldsymbol{z}) \cdot \alpha_l^{c^\nu_{\min}}$, $\kappa^\nu(P, \boldsymbol{s}, \boldsymbol{z}) \cdot \alpha_l^{c^\nu_{\max}}$. The amount of labour that ν receives is the (potentially degenerate) interval $\left[\kappa^\nu(P, \boldsymbol{s}, \boldsymbol{z}) \cdot \alpha_l^{c^\nu_{\min}}, \kappa^\nu(P, \boldsymbol{s}, \boldsymbol{z}) \cdot \alpha_l^{c^\nu_{\max}}\right]$, and so, for any $\nu \in \mathcal{N}$, if L^ν_G is more (*resp.*, less) than $\kappa^\nu(P, \boldsymbol{s}, \boldsymbol{z}) \cdot \alpha_l^{c^\nu_{\max}}$ (*resp.*, $\kappa^\nu(P, \boldsymbol{s}, \boldsymbol{z}) \cdot \alpha_l^{c^\nu_{\min}}$) then ν is regarded as 'giving' more (*resp.*, less) labour than ν receives and therefore a member of $\mathcal{N}^{ted}$ (*resp.*, $\mathcal{N}^{ter}$).

The key intuitions behind **LE** are contained in Claims 1-9, and we refer to the previous sections for a detailed discussion. Indeed, it can be shown that a definition of exploitation satisfies Claims 1-9 if and only if it satisfies **LE**. Two additional points, however, are worth making here, which highlight some further implications of Claims 1-9.

First, **LE** allows for the possibility that *all* agents in the economy be either exploited or exploiters, that is, it allows for the possibility that $\mathcal{N}^{ted} = \mathcal{N}$, or $\mathcal{N}^{ter} = \mathcal{N}$. Both cases seem rather peculiar and one may argue that they should be ruled out. However, given the nature of **LE** as a *minimum* domain condition, it is appropriate to adopt a weaker formulation. For even some of the classic definitions of exploitation - such as Morishima's [25] - do not exclude these cases. We return to this issue in the concluding section.

Second, **LE** requires that both the ERBs and their labour content be determined with reference to production, but otherwise imposes no restrictions. In UEL approaches, however, it is often required that only efficient production processes be considered in order to determine exploitation status. This is in line with Marx's notion of *socially necessary labour time*, and it is also intuitive as it seems objectionable to allow for exploitative relations emerging exclusively from productive inefficiencies and mistakes.

Let $\partial P \equiv \{\alpha \in P \mid \nexists \alpha' \in P \text{ such that } \alpha' > \alpha\}$ denote the set of efficient production activities. The next claim restricts the definition of reference labour amounts to efficient

---

[42]**LE** generalises similar axioms analysed by Yoshihara and Veneziani [54], Yoshihara [52], and Veneziani and Yoshihara [47].

processes.

Claim 10. (*Productive efficiency*) At any allocation $z \in \mathcal{Z}$, for all $\nu \in \mathcal{N}$, $\alpha^{c_{\min}^\nu} \in \partial P$ and $\alpha^{c_{\max}^\nu} \in \partial P$ with $\widehat{\alpha}^{c_{\min}^\nu} \not> c_{\min}^\nu$, $\widehat{\alpha}^{c_{\max}^\nu} \not> c_{\max}^\nu$.

By Claim 10, the ERBs should be producible as net output of efficient production activities. The notion of productive efficiency adopted is extremely weak, in that it requires that there be no activity that can produce strictly more of every output by using strictly less of every input (including labour). Claim 10 is thus quite mild and plausible, and one may argue that **LE** should be strengthened to incorporate it.[43] According to Morishima ([24], p.180), for example, in the computation of labour amounts one should focus on "those techniques by the use of which the amount of labour needed to produce given amounts of commodities can be minimized." From a normative viewpoint, these are the techniques that maximise the productivity of human labour (ibid., p.184), and "if the worker owned all the necessary means of production himself, all he would have to do would be to work only the minimum amount of hours [necessary to produce his wage-goods]" (Morishima and Catephores [26], p.41; see also Roemer [31], p.38; [32], pp.168-173). From a positive viewpoint, inefficient techniques are at best a transient epiphenomenon in a competitive economy. Yet, one may argue that if the normative emphasis of UEL theory is on participation in productive activities, then actual labour time is what matters since it "describes actual participation" (Sen [40], p.178; see also Flaschel [11, 12]). Thus, consistently with our aim of capturing the core of UEL exploitation theory, we have stated Claim 10 separately because most, but not all of the UEL approaches endorse productive efficiency.

To verify that **LE** captures the core tenets of UEL theory, we now show that all of the main definitions in the literature satisfy it. For the sake of concreteness, we shall consider allocations $z = (\Lambda, x) \in \mathcal{Z}$ with a price system (p, w, r) that are *market-feasible* in that $px^\nu = w\Lambda^\nu + rp\omega^\nu$ holds for all $\nu \in \mathcal{N}$, and if x^ν is the commodity bundle actually purchased by agent $\nu \in \mathcal{N}$ then in what follows we shall denote it as c^ν.

For any bundle $c \in \mathbb{R}_+^M$, let $l.v.(c)$ denote the *minimum* amount of (effective) labour necessary to produce c as net output.[44] In his classic definition, Morishima [25] focuses on the bundle actually purchased by an agent, c^ν, and defines its labour content as $l.v.(c^\nu)$:

---

[43]Claim 10 is indeed imposed in the formulations of **Labour Exploitation** in Yoshihara and Veneziani [54]; Yoshihara [52]; and Veneziani and Yoshihara [47].

[44]Formally, $l.v.(c) \equiv \min \{\alpha_l \mid \alpha \in \phi(c)\}$.

Definition 1 *(Morishima [25]): Consider any market-feasible allocation $\boldsymbol{z}$ with prices (p, w, r). Agent $\nu \in \mathcal{N}$, who supplies Λ^ν and purchases c^ν, is exploited if and only if $\Lambda^\nu > l.v.(c^\nu)$ and an exploiter if and only if $\Lambda^\nu < l.v.(c^\nu)$.*

Definition 1 has some desirable characteristics, according to Morishima ([25], pp.616-618): the notion of exploitation is well-defined because, under weak assumptions concerning the production set, $l.v.(c)$ is unique, well-defined and positive whenever $c \neq (0, ..., 0)$ and exploitation status is determined prior to and independent of price information, as in the standard Marxian approach, focusing only on production data.

According to Roemer [32], however, Definition 1 is conceptually flawed because it embodies a merely technological concept of value and "is independent of the social relations of production" (Roemer [32], p.152), as it identifies exploitation status based on production techniques that may never be used by profit-maximising capitalists. Like Morishima [25], Roemer [32] focuses on the bundle actually bought by agents, c^ν, but argues that its labour content should be given by the minimum amount of (effective) labour necessary to produce it as net output *among profit-rate-maximising activities at given equilibrium prices*, for only the latter production processes will be activated in a capitalist economy.

Let a market-feasible allocation $\boldsymbol{z} = (\boldsymbol{\Lambda}, \boldsymbol{x}) \in \mathcal{Z}$ with a price system (p, w, r) be *an equilibrium allocation* whenever for any $\nu \in \mathcal{N}$, (x^ν, Λ^ν) is preferred by ν to any other (x'^ν, Λ'^ν) satisfying $x'^\nu \in \mathbb{R}_+^M$, $0 \leqq \Lambda'^\nu \leqq l^\nu$, and $px'^\nu = w\Lambda'^\nu + rp\omega^\nu$.[45] For any bundle $c \in \mathbb{R}_+^M$, let $l.v.(c; p, w, r)$ be the *minimum* amount of (effective) labour necessary to produce c^ν as net output with a profit-rate-maximising activity at given equilibrium prices.[46] Then:

Definition 2 *(Roemer [32], chapter 5): Consider an equilibrium allocation $\boldsymbol{z}$ with prices (p, w, r). Agent $\nu \in \mathcal{N}$, who supplies Λ^ν and purchases c^ν, is exploited if and only if $\Lambda^\nu > l.v.(c^\nu; p, w, r)$ and an exploiter if and only if $\Lambda^\nu < l.v.(c^\nu; p, w, r)$.*

---

[45] An equilibrium allocation is essentially a *Reproducible solution* as defined by Roemer ([32], pp.64, 114). A thorough discussion of this notion goes beyond the boundaries of this paper. Intuitively, a reproducible solution involves individual optimisation, nonnegative aggregate excess demand in all markets of produced goods, and market clearing in the labour and/or credit market.

[46] Formally, let $P^\pi(p, w, r) \equiv \left\{ \alpha \in P \mid \frac{p\widehat{\alpha} - rp\underline{\alpha} - w\alpha_l}{p\underline{\alpha}} = \max_{\alpha' \in P} \frac{p\widehat{\alpha}' - rp\underline{\alpha}' - w\alpha_l'}{p\underline{\alpha}'} \right\}$ be the set of activities that maximise the rate of return on capital at prices (p, w, r). Then, $l.v.(c; p, w, r) \equiv \min \{\alpha_l \mid \alpha \in \phi(c) \cap P^\pi(p, w, r)\}$.

Although they preserve many standard insights of exploitation theory, Definitions 1 and 2 have been criticised because exploitation status depends on counterfactual amounts of labour content (Flaschel [11, 12]). For the production activities yielding $l.v.\,(c^\nu)$ or $l.v.\,(c^\nu; p, w, r)$ may be different from those actually used in equilibrium. According to critics, this use of counterfactuals is theoretically undesirable and makes exploitation an empirically vacuous notion, since the computation of $l.v.\,(c^\nu)$ and $l.v.\,(c^\nu; p, w, r)$ requires information that is not available, including, in Morishima's own words, "information about *all* the available techniques of production, actually chosen *or potentially usable*" ([25], p.617, italics added).

The third definition considered here is an extension of the so-called 'New Interpretation' (Duménil [6]; Foley [15]).[47] For any $p \in \mathbb{R}_+^M$ and $c \in \mathbb{R}_+^M$, let $\mathcal{B}\,(p, c)$ be the set of bundles that cost exactly as much as c at prices p.[48] Let $\alpha^{p,w,r} \in P$ be the *aggregate (profit-rate-maximising) production activity* at an equilibrium allocation $\boldsymbol{z}$ with prices (p, w, r).

Definition 3 : *Consider any equilibrium allocation $\boldsymbol{z}$ with prices (p, w, r) and aggregate profit-rate-maximising production activity $\alpha^{p,w,r}$. For each $c \in \mathbb{R}_+^M$ with $pc \leqq p\widehat{\alpha}^{p,w,r}$, let $\tau^c \in [0, 1]$ be such that $\tau^c\widehat{\alpha}^{p,w,r} \in \mathcal{B}\,(p, c)$. The labour content of c at $\boldsymbol{z}$ is $\tau^c\alpha_l^{p,w,r}$.*

Definition 3 identifies the labour associated with, or contained in any nonnegative bundle of goods. By Definition 3, the labour content of aggregate net output, $\widehat{\alpha}^{p,w,r}$, is equal to total social labour, $\alpha_l^{p,w,r}$, and for any bundle c whose value does not exceed national income $p\widehat{\alpha}^{p,w,r}$, the labour contained in c is equal to the fraction τ^c of social labour necessary to produce a fraction of aggregate net output, $\tau^c\widehat{\alpha}^{p,w,r}$, that has the same value as c.[49] As in Roemer's [32] approach, in Definition 3 the labour content of a bundle can be identified only if the price vector is known. Yet social relations play a more central role, because the definition of labour content requires a prior knowledge of the social reproduction point $\alpha^{p,w,r}$ and labour content is explicitly linked to the redistribution of total social labour. Then:

Definition 4 *(New Interpretation): Consider any equilibrium allocation $\boldsymbol{z}$ with prices (p, w, r) and aggregate profit-rate-maximising production activity $\alpha^{p,w,r}$. For any $\nu \in \mathcal{N}$, who supplies Λ^ν and purchases c^ν, let τ^{c^ν} be defined as in Definition 3. Agent ν is exploited if and only if $\Lambda^\nu > \tau^{c^\nu}\alpha_l^{p,w,r}$ and an exploiter if and only if $\Lambda^\nu < \tau^{c^\nu}\alpha_l^{p,w,r}$.*

---

[47]See Yoshihara and Veneziani [54]; Yoshihara [52]; Veneziani and Yoshihara [47, 46]

[48]Formally: $\mathcal{B}\,(p, c) \equiv \left\{ y \in \mathbb{R}_+^M \mid py = pc \right\}$.

[49]If $p\widehat{\alpha}^{p,w,r} = 0$, then τ^c is actually undetermined, but in (rather special) equilibria in which the value of aggregate net output is zero, it seems reasonable to impose that $\tau^c = 0$.

In other words, for any agent $\nu \in \mathcal{N}$, τ^{c^ν} represents ν's share of national income, and so $\tau^{c^\nu} \alpha_l^{p,w,r}$ is the share of social labour that ν receives by earning income barely sufficient to buy c^ν. Thus, in the New Interpretation, the notion of exploitation is related to the production and distribution of national income and social labour, and depends on empirically observable magnitudes. Yet, Definition 4 has been criticised because, unlike Definitions 1 and 2, the actual bundles purchased by agents are only indirectly relevant to determine exploitation status, and unlike Definition 1, the notion of exploitation depends on price information.

In summary, various definitions can be, and have in fact been, proposed. They have different normative and positive implications, and yet all of them share the fundamental insights of UEL exploitation theory as stated in Claims 1-9. Indeed, they all satisfy **LE**.

To see this, note that in all of the three approaches examined, for all $\nu \in \mathcal{N}$, $L_G^\nu = \Lambda^\nu$ and $\kappa^\nu (P, \boldsymbol{s}, \boldsymbol{z}) = 1$, and the ERB is unique. In Definitions 1 and 2, the ERB corresponds to the bundle actually purchased by agents $c_{\min}^\nu = c_{\max}^\nu \equiv c^\nu \in B(\omega^\nu, l^\nu; p, w, r)$. However, in Definition 1, the labour content of the ERB is the minimum amount of labour necessary to produce it among all conceivable production activities, whereas in Definition 2 it is the minimum amount of labour necessary to produce the ERB among all profit rate maximising processes.[50] In Definition 4, given an equilibrium allocation $\boldsymbol{z}$ with prices (p, w, r) and aggregate profit-rate-maximising production activity $\alpha^{p,w,r}$, the ERB is defined counterfactually as the proportion of net aggregate output that the agent may purchase with her income $c_{\min}^\nu = c_{\max}^\nu = c_e^\nu \equiv \tau^{c^\nu} \widehat{\alpha}^{p,w,r} \in B(\omega^\nu, l^\nu; p, w, r)$, where $\tau^{c^\nu} = \frac{pc^\nu}{p\widehat{\alpha}^{p,w,r}}$, and its labour content is given by the corresponding share of aggregate social labour, $\alpha^{c_e^\nu} \equiv \tau^{c^\nu} \alpha^{p,w,r}$.

As in Yoshihara [52], it can also be shown that Definitions 1 and 2 would satisfy **LE** if they were reformulated to be independent of agents' consumption choices - as suggested by Roemer ([32], chapter 4) - by focusing, for example, on the *maximum* and the *minimum* amounts of labour that agents can receive with their income so that $c_{\min}^\nu, c_{\max}^\nu \in B(\omega^\nu, l^\nu; p, w, r)$ but in general $c_{\min}^\nu \neq c_{\max}^\nu \neq c^\nu$. Similarly, it would be straightforward - albeit notationally intensive - to show that other definitions satisfy **LE**, including the input-output approach proposed by Flaschel [11, 12]; the variant of Definition 2 that focuses on *any* activities in $\phi(c) \cap P^\pi(p, w, r)$, and not only labour-minimising ones (Roemer [32], pp.164-168); and even the subjectivist approach developed by Matsuo [22], according to which the labour contained

---

[50]Formally: in Definition 1, $\alpha^{c^\nu} \in \arg\min\{\alpha_l \mid \alpha \in \phi(c^\nu)\}$; in Definition 2, $\alpha^{c^\nu} \in \arg\min\{\alpha_l \mid \alpha \in \phi(c^\nu) \cap P^\pi(p, w, r)\}$.

in a vector c corresponds to the minimum amount of labour necessary to produce another bundle c' as net output, which gives at least as much utility as c.[51]

This confirms that **LE** incorporates the main insights of UEL exploitation theory shared by *all* of the main approaches.

6   Refinements

The purpose of the previous analysis is primarily descriptive: Claims 1-9 aim to disentangle and clarify the intuitions behind all UEL approaches. Despite the seemingly irreconcilable differences between definitions focusing on physical or monetary magnitudes, actual choices or potential consumption, objective data or subjective preferences, actual or counterfactual production processes, and so on, **LE** identifies - at a deep conceptual level - a fundamental structure that defines UEL exploitation theory in *all* of its variants - even though this structure, and the notions of ERBs and their labour content are usually left implicit.

By capturing the foundational intuitions common to all UEL approaches, Claims 1-9 can be interpreted as defining the boundaries of the *admissible class* of UEL exploitation forms and **LE** can be seen as a domain axiom that all UEL definitions should satisfy. From this perspective, Claims 1-9 at the same time identify the common foundations of UEL theory *and* provide the formal and conceptual framework to examine the normative and positive implications of different definitions.[52] For the boundaries of UEL theory identified by **LE** are extremely wide and potentially include an infinite number of conceivable definitions - i.e. an infinite number of ways of specifying the ERBs, the reference labour amounts, and L_G^{ν}. In this section, we illustrate this point by showing how the differences between alternative definitions concerning two important aspects of UEL exploitation can be conceived of as different refinements of, or restrictions on **LE**.

A first important issue concerns the relevance of individual choices in determining exploitation status. In **LE**, the amount of labour that agent ν receives is determined by some reference bundle(s) that ν can in principle purchase. In the standard approach, however, the ERB is the bundle c^{ν} actually chosen by agent ν at allocation $\boldsymbol{z}$. Thus, the standard view

---

[51] Provided the same (representative) utility function is used for all agents, or else Claim 9 is violated.

[52] For example, Flaschel's ([11], pp.440-1, 449 and [12], pp.17-20) discussion of the desirable properties of the concept of labour values can be interpreted as identifying a set of refinements of Claims 7-9.

can be captured in the following additional restriction:

Claim 11a. (*Consumer subjectivism*) At any allocation $z \in \mathcal{Z}$, $c_{\min}^{\nu} = c_{\max}^{\nu} = c^{\nu}$ for all agents $\nu \in \mathcal{N}$.

The theoretical justification for Claim 11a rests on the idea that UEL exploitation status should be defined based only on *actual* economic data. Disregarding agents' consumption choices implies identifying their exploitation status based on information that is potentially at odds with their actual situation and well-being.[53]

This subjectivist view is not uncontroversial. Following the standard Marxian approach, for example, one may insist that exploitation status depend on *production* decisions, and not on possibly idiosyncratic *consumer* choices. From this viewpoint, agents who are identical in all characteristics, except possibly in their consumption choices, should have the same exploitation status. This view can be formalised in the following Claim:

Claim 11b. (*Consumer objectivism*) At any allocation $z \in \mathcal{Z}$, for any two agents $\nu, \mu \in \mathcal{N}$, if $B(\omega^{\nu}, l^{\nu}; p, w, r) = B(\omega^{\mu}, l^{\mu}; p, w, r)$ then $c_{\min}^{\nu} = c_{\min}^{\mu}$ and $c_{\max}^{\nu} = c_{\max}^{\mu}$.

Claim 11b is called 'Consumer objectivism' because it does not rule out the possibility that subjective factors matter in other aspects of UEL exploitation theory - for example, by defining L_G^{ν} as the agents' actual labour supply or even by introducing subjective elements in the determination of the labour 'content' of the ERBs (as in Matsuo [22], see above).

Next, note that in our previous axiomatic analysis, we have left L_G^{ν} unspecified because there is no widely shared view on the appropriate definition of the labour 'given' by agents in general economies with heterogeneous labour. In particular, two main approaches can be distinguished which focus either on the *effective* labour contributed, or on the labour *time* expended by agents. The former approach can be formalised as follows:

Claim 12a. (*The contribution view*) At any allocation $z \in \mathcal{Z}$, the amount of labour that each agent 'gives' is the effective labour that she contributes in economically relevant activities, namely $L_G(z) = \mathbf{\Lambda} = (\Lambda^1, ..., \Lambda^N)$. Correspondingly, for all $\nu \in \mathcal{N}$, $\kappa^{\nu}(P, s, z) = 1$ and the reference labour amounts are $L_{\min}^{\nu} = \alpha_{l\min}^{\nu}$ and $L_{\max}^{\nu} = \alpha_{l\max}^{\nu}$.

---

[53]One may also support Claim 11a on the grounds of consistency if L_G^{ν} is equal to actual (effective) labour supply. For, "If we wish to define exploitation with reference to the labor an agent actually expends ... then we must admit the preference-relatedness of exploitation in any case" (Roemer [32], p.133).

By Claim 12a, both labour 'given' and labour 'received' are measured in terms of effective labour. This is the natural extension of all of the classic definitions of exploitation in the Okishio-Morishima-Roemer tradition, as noted in section 5 above, and it is the standard approach in the literature on exploitation in economies with heterogeneous labour (see, e.g., Krause [20]; Duménil *et al* [7]). Moreover, the '*contribution view*' incorporates an important normative intuition: an efficient and UEL exploitation-free allocation coincides with the *proportional solution*, a well-known fair allocation rule whereby every agent's income is proportional to her contribution to the economy (Roemer and Silvestre [37]). Proportionality is a strongly justified normative principle, whose philosophical foundations can be traced back to Aristotle (Maniquet [21]), and it can be justified in terms of the Kantian categorical imperative (Roemer [36]).[54]

Alternatively, one may argue that UEL exploitation theory captures some inequalities in the distribution of material well-being and free hours that are - at least *prima facie* - of normative relevance (Fleurbaey [13, 14]).[55] For example, they may be deemed relevant because material well-being and free hours are two key determinants of *individual well-being freedom* (Rawls [27]; Sen [41]). But they are also relevant in approaches that link exploitation and the Marxian notion of alienation in production (Buchanan [2]). From this perspective, the key variable of normative interest is labour *time*:[56]

Claim 12b. (*The well-being view*) At any allocation $z \in \mathcal{Z}$, the amount of labour that each agent 'gives' is the labour time that she spends in economically relevant activities, namely $L_G(z) = \lambda(z) = (\lambda^1, ..., \lambda^N)$. Correspondingly, the reference labour amounts for each agent $\nu \in \mathcal{N}$ are $L_{\min}^\nu = \kappa^\nu(P, s, z) \cdot \alpha_{l\min}^\nu$ and $L_{\max}^\nu = \kappa^\nu(P, s, z) \cdot \alpha_{l\max}^\nu$.

In Claim 12b, the positive linear transformations $(\kappa^\nu)_{\nu \in \mathcal{N}}$ are necessary to transform units of effective labour into labour *time*, and are left undefined as there are in principle various theoretically relevant specifications. For example, suppose that $L_R(z)$ is assumed to capture

---

[54] The contribution principle ('To each according to his contribution') is also one of the principles of justice analysed by Marx in the *Critique of the Gotha programme* (Husami [19]; Miller [23]; Cohen [4]; Warren [49]).

[55] Cohen ([4], p.203) aptly refers to differences in the "leisure-and-income sets" available to agents. Roemer ([32], p.75 and passim) talks about the "exploitation-welfare criterion".

[56] Claim 12b holds also in the approach proposed by Arneson ([1], pp.212-3) whereby exploitative relations violate the principle that "Those who make equal productive sacrifices are equally deserving of economic remuneration". Desert is measured against the *time* (and effort) spent in production, while discounting natural abilities and skills as morally arbitrary. See also Elster ([8], p.202) and Reiman ([29], pp.9ff).

the distribution of social labour and let $\alpha^{c_{\min}^\nu} \in \phi\left(c_{\min}^\nu\right)$ and $\alpha^{c_{\max}^\nu} \in \phi\left(c_{\max}^\nu\right)$ be the outcomes of *social* production activity. Then, for all $\nu \in \mathcal{N}$, it seems natural to define $\kappa^\nu\left(P, \boldsymbol{s}, \boldsymbol{z}\right)$ as the average skill level at allocation $\boldsymbol{z}$: $\kappa^\nu\left(P, \boldsymbol{s}, \boldsymbol{z}\right) \equiv \kappa\left(\boldsymbol{s}, \boldsymbol{z}\right) \equiv \frac{\sum_{\nu \in \mathcal{N}} \lambda^\nu}{\sum_{\nu \in \mathcal{N}} s^\nu \lambda^\nu}$. Alternatively, one may argue that $L_R(\boldsymbol{z})$ should be equal to the labour time that would be necessary for each agent to produce the ERBs *autarkically*. In this case, one should set $\kappa^\nu\left(P, \boldsymbol{s}, \boldsymbol{z}\right) \equiv \frac{1}{s^\nu}$, for all $\nu \in \mathcal{N}$.

In economies with homogeneous labour, the contribution view and the well-being view coincide but this is not generally true, and different approaches will have rather different normative and positive implications. As the previous discussion shows, our analysis provides the formal and conceptual framework to rigorously examine these differences.

7 Conclusions

This paper has laid out the foundations of UEL exploitation theory by rigorously formulating and examining the intuitions common to all approaches. We have first identified the basic conceptual structure of UEL theory, including the notions of exploitation reference bundles and the labour associated with them. Then, we have formulated its core intuitions as a series of formally weak and theoretically robust Claims, collected in axiom **LE**. We have shown that our analysis is descriptively accurate: *all* of the main approaches indeed satisfy axiom **LE** (and therefore Claims 1-9). Finally, we have shown that our analysis, and in particular the domain axiom **LE**, provides the framework to analyse various normatively relevant issues, and can shed light on the differences between UEL exploitation theory and other approaches, as well as on the differences between alternative UEL definitions.

To be sure, this descriptive exercise does not provide an answer to the issue of the normative relevance of UEL exploitation theory, or a justification for the focus on labour as the variable of normative concern. Nonetheless, a clear statement of the foundations of UEL theory represents a fundamental step to tackle these issues. Indeed, this paper provides a novel, rigorous axiomatic framework in which to address them and to identify an appropriate definition of exploitation starting from first principles. For, if **LE** defines the domain of admissible UEL exploitation forms, then one may try to identify the normatively relevant properties that a definition should satisfy and that perhaps uniquely characterise one definition *within the admissible set*. From this perspective, some central insights of exploitation

theory, such as the existence of a relation between class and exploitation status (Roemer's [32] celebrated *Class-Exploitation Correspondence Principle*) or the correspondence between positive profits and the exploitation of at least the poorest segments of the working class (see, for example, the *Profit-Exploitation Correspondence Principle* recently proposed by Veneziani and Yoshihara [47, 46]) may be reformulated as axiomatic requirements.

To be sure, both the equivalence between positive profits and the existence of exploitation, and the correspondence between class and exploitation status have usually been considered as theoretical *results*, which hold under some conditions but not others.[57] Nonetheless their central relevance in UEL exploitation theory is such that their epistemological status is as postulates: they are properties that any UEL definition should satisfy and alternative definitions have often been proposed and compared in the literature based on whether they preserve them (see, for example, Roemer [32], pp.148-153).

Similarly, one may argue that an appropriate definition of exploitation should rule out the possibility that at a given allocation all agents be exploiters or exploited (as noted in section 5 above, **LE** allows for this possibility). One way to do so is by explicitly formalising the crucial relational aspect inherent in exploitative relations, such that if an agent is exploited, she must be exploited *by someone*, and vice versa if an exploiter exists, she must be exploiting *someone*, as suggested by Yoshihara and Veneziani [54]:

Relational Exploitation (RE): *At any equilibrium allocation $z \in \mathcal{Z}$, the two subsets $\mathcal{N}^{ter}$ and $\mathcal{N}^{ted}$ are such that $\mathcal{N}^{ter} \neq \varnothing$ if and only if $\mathcal{N}^{ted} \neq \varnothing$.*

Within the domain identified by **LE**, are there any definitions that capture important normative intuitions of UEL exploitation theory, such as the correspondence between class and exploitation status, the relation between exploitation and profits, and **RE**? If not, and an impossibility result followed from the imposition of these properties, this would arguably raise serious questions about some of the key intuitions of UEL theory, or indeed about the viability of the UEL approach itself.[58]

Some preliminary results by Yoshihara and Veneziani [54, 52, 47, 46] suggest that the set of definitions that satisfy **LE** and these additional properties is actually non-empty and,

---

[57]As in the classic literature on the so-called *Fundamental Marxian Theorem*. See, among the many others, Morishima [25]; Roemer [31]; Krause [20], Fleurbaey [13]; Flaschel [11, 12]; Yoshihara [53].

[58]For the opposite view, and a rejection of the intuition behind **RE**, see Roemer ([32], chapter 4).

among all of the main approaches, the only one satisfying all properties is Definition 4: if either Definition 1 or Definition 2 is adopted, there are allocations in which **RE** is violated and *all* agents are exploited; and there are allocations in which profits are positive but no propertyless worker is exploited.

Although these results are preliminary and hold in specific economic settings, they do suggest that the axiomatic road investigated in this paper is both insightful and promising.

References

[1] Arneson, R.J., 1981. What's Wrong with exploitation? *Ethics* 91, 202-227.

[2] Buchanan, A.E., 1982. *Marx and Justice*. Rowman and Littlefield, Totowa.

[3] Cohen, G.A., 1979. The Labor Theory of Value and the Concept of Exploitation. *Philosophy & Public Affairs* 8, 338-360.

[4] Cohen, G.A., 1995. *Self-Ownership, Freedom and Equality*. Cambridge University Press, Cambridge.

[5] Desai, M., 1979. *Marxian Economics*. Rowman and Littlefield, Totowa.

[6] Duménil, G., 1980. *De la Valeur aux Prix de Production*. Economica, Paris.

[7] Duménil, G., Foley, D.K., Lévy, D., 2009. A Note on the Formal Treatment of Exploitation in a Model with Heterogeneous Labor. *Metroeconomica* 60, 560-567.

[8] Elster, J., 1985. *Making Sense of Marx*. Cambridge University Press, Cambridge.

[9] Elster, J., 1997. Exploitation, Freedom, and Justice. In: Nielsen, K., Ware, R., (eds.) *Exploitation*. Humanities press, New Jersey.

[10] Emmanuel, A., 1972. *Unequal Exchange*. Monthly Review Press, New York.

[11] Flaschel, P., 1983. Actual Labor Values in a General Model of Production. *Econometrica* 51, 435-454.

[12] Flaschel, P., 2010. *Topics in Classical Micro- and Macroeconomics*. Springer, New York.

[13] Fleurbaey, M., 1996. *Théories Économiques de la Justice*. Economica, Paris.

[14] Fleurbaey, M., 2014. The Facets of Exploitation. *Journal of Theoretical Politics* 26, 653-676.

[15] Foley, D.K., 1982. The Value of Money, the Value of labour Power, and the Marxian Transformation Problem. *Review of Radical Political Economics* 14, 37-47.

[16] Goodin, R., 1987. Exploiting a situation and exploiting a person. In: Reeve, A. (ed.) *Modern theories of exploitation*. Sage, London.

[17] Holländer, H., 1982. Class Antagonism, Exploitation and the Labour Theory of Value. *Economic Journal* 92, 868-885.

[18] Holmstrom, N., 1977. Exploitation. *Canadian Journal of Philosophy* 7, 353-369.

[19] Husami, Z.I., 1978. Marx on Distributive Justice. *Philosophy & Public Affairs* 1, 27-64.

[20] Krause, U., 1982. Heterogeneous Labour and the Fundamental Marxian Theorem. *Review of Economic Studies 48*, 173-178.

[21] Maniquet, F., 2002. A study of proportionality and robustness in economies with a commonly owned technology. *Review of Economic Design* 7, 1-15.

[22] Matsuo, T., 2008. Profit, Surplus Product, Exploitation and Less than Maximized Utility. *Metroeconomica* 59, 249-265.

[23] Miller, D., 1990. *Market, State and Community*. Oxford University Press, Oxford.

[24] Morishima, M., 1973. *Marx's economics*. Cambridge University Press, Cambridge.

[25] Morishima, M., 1974. Marx in the Light of Modern Economic Theory. *Econometrica* 42, 611-632.

[26] Morishima, M., Catephores, G., 1978. *Value, Exploitation and Growth*. McGraw Hill, London.

[27] Rawls, J., 1971. *A Theory of Justice*. Harvard University Press, Cambridge, MA.

[28] Reiff, M.R., 2013. *Exploitation and Economic Justice in the Liberal Capitalist State*. Oxford University Press, Oxford.

[29] Reiman, J., 1987. Exploitation, Force and the Moral Assessment of Capitalism: Thoughts on Roemer and Cohen. *Philosophy & Public Affairs* 16, 3-41.

[30] Robinson, J., 1933. *Economics of Imperfect Competition.* London, Macmillan.

[31] Roemer, J.E., 1981. *Analytical foundations of Marxian economic theory.* Harvard University Press, Cambridge, MA.

[32] Roemer, J.E., 1982. *A General Theory of Exploitation and Class.* Harvard University Press, Cambridge, MA.

[33] Roemer, J.E., 1982. Property relations vs. in Marxian exploitation. *Philosophy & Public Affairs* 11, 281-313.

[34] Roemer, J.E., 1985. Should Marxists be interested in exploitation? *Philosophy & Public Affairs* 14, 30-65.

[35] Roemer, J.E., 1989. What is exploitation? Reply to Jeffrey Reiman. *Philosophy & Public Affairs* 18, 90-97.

[36] Roemer, J.E., 2010. Kantian equilibrium. *Scandinavian Journal of Economics* 112, 1-24.

[37] Roemer, J.E., Silvestre, J.E., 1993. The proportional solution for economies with both private and public ownership. *Journal of Economic Theory* 59, 426-444.

[38] Sample, R., 2003. *Exploitation: What it is and why it's wrong.* Rowman and Littlefield, Boulder.

[39] Schweickart, D., 1991. The Politics and Morality of Unequal Exchange. *Economics and Philosophy* 7, 13-36.

[40] Sen, A.K., 1978. On the labour theory of value: some methodological issues. *Cambridge Journal of Economics* 2, 175-190.

[41] Sen, A.K., 1985. *Commodities and Capabilities.* North-Holland, Amsterdam.

[42] Steedman, I., 1977. *Marx after Sraffa.* New Left Books, London.

[43] Steiner, H., 1984. A liberal theory of exploitation. *Ethics* 94, 225-241.

[44] Veneziani, R., 2007. Exploitation and Time. *Journal of Economic Theory* 132, 189-207.

[45] Veneziani, R., 2013. Exploitation, Inequality, and Power. *Journal of Theoretical Politics* 25, 526-545.

[46] Veneziani, R., Yoshihara, N., 2014. One million miles to go: taking the axiomatic road to defining exploitation. Mimeo, QMUL and Hitotsubashi University.

[47] Veneziani, R., Yoshihara, N., 2015. Exploitation in economies with heterogeneous preferences, skills and assets: An axiomatic approach. *Journal of Theoretical Politics* 27, 8-33.

[48] Vrousalis, N., 2013. Exploitation, Vulnerability, and Social Domination. *Philosophy and Public Affairs* 41, 131-157.

[49] Warren, P., 1997. Should Marxists Be Liberal Egalitarians? *Journal of Political Philosophy* 5, 47-68.

[50] Wood, A.W., 1995. Exploitation. *Social Philosophy and Policy* 12, 135-158.

[51] Wright, E.O., Levine, A. and Sober, E. 1992. *Reconstructing Marxism*, Verso, London.

[52] Yoshihara, N., 2010. Class and Exploitation in General Convex Cone Economies. *Journal of Economic Behavior & Organization* 75, 281-296.

[53] Yoshihara, N., 2016. A Progress Report on Marxian Economic Theory: On the Controversies in Exploitation Theory since Okishio (1963). *Journal of Economic Surveys*, forthcoming.

[54] Yoshihara, N., Veneziani, R., 2009. Exploitation as the Unequal Exchange of Labour: An Axiomatic Approach. Working paper N.655, Queen Mary University of London.

[55] Yoshihara, N., Veneziani, R., 2011. Strong Subjectivism in the Theory of Exploitation: A Critique. *Metroeconomica* 62, 53-68.

School of Economics and Finance

Queen Mary
University of London