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Giulio Fella, Paola Manzini and Marco Mariotti

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Queen Mary
University of London

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Giulio Fella†

Queen Mary, University of London

Paola Manzini‡

Queen Mary, University of London and IZA

Marco Mariotti§

University of Exeter

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Abstract

In this paper we derive an explicit model of negotiations between spouses when utility is (partially) transferable only in case of separation. We show that inefficient separation may occur in equilibrium even under consensual divorce law. This provides theoretical support for the view that changes in social norms rather than in legislation may be responsible for increasing divorce rates.

Keywords: Bargaining, divorce, non transferability

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†Department of Economics, Queen Mary, University of London, Mile End Road, London E1 4NS (tel: 020-7882 5088; e-mail: g.fella@qmul.ac.uk, http://www.qmw.ac.uk/~ugte176).

‡Department of Economics, Queen Mary, University of London, Mile End Road, London E1 4NS (tel: 020-7882 5083; e-mail: p.manzini@qmul.ac.uk, http://www.qmw.ac.uk/~ugte172).

§Department of Economics, University of Exeter, Streatham Court, Rennes Drive, Exeter EX4 4PU, UK (tel: 01392-263219; e-mail: m.mariotti@exeter.ac.uk).

1 Introduction

The transition from a "fault" to a "no-fault" divorce is often blamed for increases in divorce rates. For example, the divorce legislation that began in most US states around 1970 has been followed over the last thirty years by a large increase in divorce rates. However, the causal relation between marriage laws and marital separation is far from clear, and the empirical evidence is not conclusive[1]. In this paper we develop a theoretical model to study this issue, and we uncover some interesting and unexpected effects of marriage law. Our approach is based on the idea that the extent to which marital assets (and thus utility) are transferable between partners is a crucial determinant of divorce behaviour.

The reason why transferability plays a central role is that, from a theoretical perspective, a change in the law from consensual to unilateral divorce[2] amounts to a reallocation of the property rights on the marriage away from the spouse who does not want the marriage to end. Assuming full transferability, the economic analysis of marriage breakdown has traditionally been carried out in the shadow of the Coase theorem. In their seminal paper Becker, Landes and Michael (1977) argue that

> "If all compensations between spouses are feasible and costless then separation takes place when combined wealth from divorce is higher than from marriage."

Thus, within such a framework separation is always efficient, in the sense that it maximises joint welfare. Divorce law may only affect the distribution of the gains from staying together or separate, but not the probability of marriage breakdown, *provided that* utility is freely transferable *both* within the marriage *and* in case of separation, and barring other transaction costs.

[1]Peters (1986, 1992), Johnson and Skinner (1986), Weiss and Willis (1995) and Gray (1998) find that changes in divorce legislation did not significantly affect marital stability. On the other hand, Allen (1992), Zelder (1993) and Johnson and Skinner (1986) reach the opposite conclusion. Though Friedberg (1998) has convincingly argued that the result in Peters (1986) may be biased downward due to the omission of state-specific time trends which are positively correlated with changes in legislation. Gray (1998) is immune to such a critique in so far as he uses differences between two years to eliminate (linear) time-trends.

[2]While the law distinguishes between "fault" and "no-fault" divorce, the relevant economic categories are consensual versus non-consensual. The two concepts are not exactly equivalent. In what follows we will abstract from such differences.

Although the transferable utility case is a crucial benchmark, in practice there are important elements of non-transferability within the marriage, as a large component of consumption when married is joint. This is the case, for example, with children and the services of some owned assets, such as the family home. Conversely, once the marriage is dissolved, such items become transferable through the usual solutions to convert indivisible common property into a divisible commodity, such as monetization by sale, rotation (time-sharing) or randomization. If utility is non-transferable within the marriage the rate of marriage breakdown is inefficiently high under unilateral divorce as, even if separation reduces joint wealth, it is possible that the spouse who would like the relationship to continue may be unable to compensate the one who prefers to walk out. Zelder (1993) has argued that in such a case consensual divorce, by forbidding unilateral termination, obliges the spouse who wants to separate to compensate the other partner to obtain her agreement on termination and restores efficiency of separation.

In this paper we derive an explicit model of negotiations between spouses when utility is transferable only in case of separation. Disposal of a couple's jointly-enjoyed assets is the only possible way to transfer, perhaps only partially, the other components of utility. Such assets can be liquidated and thus rendered transferable only through separation. Our basic model can be taken to describe negotiations between two spouses involved in a major decision (such as having a child or relocating) which, provided both partners agree to it, generates a certain - non-transferable - marital surplus. If the couple reaches an agreement we say that they enjoy a "cooperative" marriage. Alternatively they can agree to divorce and split the transferable assets. Negotiations take place according to a variant of Rubinstein (1982) bargaining game. In each round one of the spouses can propose either the marital agreement or divorce and a partition of the transferable assets. Under consensual divorce legislation separation can take place only if both parties agree to it. A protracted disagreement does not lead to dissolution of the marriage, but prevents the parties from enjoying both the surplus from a cooperative marriage and the surplus from divorce. Perpetual disagreement within the marriage, then, is worse than either agreement or divorce.

We show that under consensual divorce an efficient marriage does not necessarily survive in equilibrium. There are two possible equilibria. In one, the efficient survival is

guaranteed. In the other, inefficient separation takes place. As under unilateral divorce, in such equilibrium the spouse who gains from divorce is able to obtain it even if it makes the other partner strictly worse off. Even more strikingly, it is possible that in the latter equilibrium it is the partner who would have preferred a cooperative continuation to unilateral separation that bribes the other spouse to obtain termination. The multiplicity stems from the fact that each spouse can credibly threaten to lock the other spouse in a non-cooperative marriage, unless his/her preferred outcome is agreed upon. This threat is instead empty under unilateral divorce, since in that case the partner who benefits from separation can walk out. A fuller intuition for these results is given in the example in section 2. The same results go through if the model is extended to the case where the surplus from a cooperative marriage does not require agreement on any major decision, but just refraining from disrupting the marriage by asking for divorce.

From a positive point of view, our result implies that even if the "no-fault" revolution did cause the observed increase in the rate of marriage breakdown, there is no reason to expect that going back to fault divorce would bring the divorce rate down. It is well possible that the change in social norms that has brought forward the no-fault revolution may imply that the inefficient equilibrium prevails under consensual divorce.

From a normative perspective our result suggests that, if utility cannot be freely transferred within the marriage, the only sure way to affect the separation decision is not by reintroducing fault divorce but by altering the returns to divorce. This is clearly problematic in so far as third parties cannot observe the spouses gains and losses from separation.

A number of papers are related to this work. Lundberg and Pollack Lundberg and Pollack (1993) first pointed out that disagreement *within* the marriage is one possible alternative to cooperation. They endogenize disagreement payoffs in the axiomatic Nash bargaining solution as a Cournot equilibrium and show how, differently from the divorce threat models of Manser and Brown (1980) and McElroy and Horney (1981), policies that transfer resources towards one or the other spouse may have distributional implications for existing marriages. Their model assumes full transferability and does not explicitly consider the possibility of divorce. We go one step further by arguing that, in a strategically founded model and under consensual divorce, disagreement within the marriage is

the only alternative to reaching agreement on separation or on a cooperative marriage.

The focus on negotiations distinguishes our approach from others which also highlight the importance of joint consumption within the marriage, such as Zelder (1993) and Chiappori and Weiss (2001). As noted above, Zelder (1993) first suggested that non-transferability implies inefficient separation under at-will, but not under consensual divorce. He exploits this prediction to test the efficient separation hypothesis. Chiappori and Weiss (2001), on the other hand, study the efficiency of the decentralized equilibrium when the matching market is characterized by search frictions. Peters (1986) was the first to analyse divorce decisions in the presence of transaction costs, the latter taking the form of asymmetric information about each partner's respective payoffs. Applying the analysis of Hall and Lazear (1984) and Hashimoto and Yu (1980) to marriage, she argued that a non-renegotiated, fixed wage contract might minimize inefficient separation. This kind of approach though is open to the criticism that suppression of renegotiation is neither necessarily efficient nor enforceable in a situation, such as marriage, where explicit wage contracts are not observed and where all kinds of possible strategic behaviour are possible. Also, such a framework implies that the parties should want to avoid negotiation altogether, not only over the terms of continuation. This is inconsistent with the fact that negotiation over the terms of termination is exactly what divorce lawyers devote a lot of time to.

Clark (1999) criticises the Coase-based view that 'divorce laws do no matter'. He, too, models negotiations within the marriage as a problem of reaching an agreement within two sets of possible payoff combinations: those associated with the surplus from staying married and the surplus from divorcing, respectively. He argues that if the efficient frontiers associated with the two sets of payoffs intersect, divorce law does in fact matter and that consensual divorce law eliminates all separations that are not Pareto improving. Unlike our paper, he does not use an explicit model of negotiations and assumes that dissolution laws determine which of the two bargaining sets can be vetoed by one of the two parties. Our result shows that his conclusions apply to just one of two possible equilibria under consensual divorce.

2 An example

Consider a married couple, Anthony and Betty. At any moment they are faced with the alternative choices of whether to continue together or separate and enjoy their outside opportunities[3]. Assuming linear utilities, let us normalize to 1 the sum of the utilities each of them derives in case they split. Let x and $(1-x)$ be Anthony's and Betty's shares, respectively, of this total in case either of them can file for "no-fault" or unilateral divorce. This corresponds to a point on the efficient frontier AB in figure 1. While such partition captures the payoff consequences of divorce, which depend both on outside opportunities and courts' decisions on compensation, the crucial and invariant feature of no-fault divorce is that it establishes the right to unilaterally walk out of the relationship. That is, with the exception of the reallocation of common property and children, it allows whichever haggling may take place over the terms of divorce to be conducted under either partner's preferred circumstances; e.g. while cohabiting with a new parner. For simplicity, let us assume that the joint utility from separation is freely transferable (e.g. it is associated with both Anthony and Betty finding two new partners with "deep pockets").

Under "fault" (consensual) divorce, instead, haggling over separation cannot be conducted under either partner's preferred circumstances. The partner's consent must be obtained in order to be able to enjoy outside opportunities without this hinging negatively on the divorce outcome. That is, haggling over the outcome has to take place, *within* the marriage. Suppose that in such circumstances the equilibrium divorce agreement would give each partner a payoff of 0.5 (point C in figure 1).

[**Figure 1 here**]

Consider now the alternative choice of staying married. Under our maintained assumption that utility is non-transferable within the marriage, the couple's payoff from the marriage is a utility pair $\{u_A^m, u_B^m\}$, a point in figure 1. Separation is efficient when the utility pair associated with the marriage lies strictly inside area AOB, e.g. point

[3]The two are not completely exclusive options. Yet, in practice, under fault divorce betraying one's spouse is not only ground for divorce, but is also likely to affect negatively the divorce outcome for the spouse at fault.

P_1. Given that all points in area AOB are feasible and P_1 is inefficient, the parties separate under either consensual or unilateral divorce. Consensual divorce just alters the distribution of the payoff from separation.

Let us now see what happens instead when continuation of the marriage is efficient. To isolate the pure allocational effect of the institutional set up, assume $x = 0.5$. If the couple separates, the partition of the joint payoff is the same - point C - under fault and no-fault divorce. In either case, Anthony and Betty would stay married if this gives them both a higher utility than C. This is the case if P_i, e.g. P_2, lies to the north-east of point C.

Alternatively continuation of the marriage does not Pareto dominate separation. For example, at point P_3 Betty prefers continuation of the marriage to separation while Anthony's ordering is the opposite. Under unilateral divorce Anthony would be free to (inefficiently) terminate the relationship and obtain a higher payoff of 0.5. Under consensual divorce survival of the marriage is an equilibrium. Indeed, it would be the only equilibrium if utility were transferable within the marriage too. The main result of this paper is that when utility is non-transferable within the marriage efficient survival of the marriage is not the only equilibrium. The inefficient outcome C is also an equilibrium even if divorce can only be consensual. The intuition is that, contrary to what is usually assumed, the utility pair P_3 is not an option that either spouse can unilaterally exercise. Enjoyment of P_3 requires both spouses to agree to it. The surplus associated with P_3 is available if the parties agree on a cooperative marriage (e.g. either party refrains from destroying by daily arguing or by refusing to agree on, for example, having a child or relocating). The coexistence of the two equilibria stems from the fact that, under consensual divorce, either party can delay/reduce enjoyment of the surplus associated with either alternative.

To consider the effect of divorce legislation on the divorce rate assume that the respective payoffs from a cooperative marriage normalized by the joint payoff from separation are random variables with a given joint distribution. Under unilateral divorce the probability of marriage survival is $\Pr\{u_A^m \geq x, u_B^m \geq 1 - x\}$, the probability that both spouses are better off in a cooperative marriage. This is the probability mass associated with the area to the north-east of point C in figure 1. Suppose the inefficient equilibrium prevails

in case of consensual divorce. Then the probability of marriage survival - the probability that for given joint payoff from separation, the utility pair associated with the marriage lies to the north-east of C - is the same as under unilateral divorce.

This result indicates that studies of the effect of changes in divorce legislation on divorce rates may shed little light on whether separations maximizes joint wealth or not. Furthermore, it also implies that divorce legislation, though not court rulings on spouse compensation, may have little effect on divorce rates.

In general, since the conditions under which haggling over separation takes place are different between unilateral and consensual divorce, the distribution of the joint return from separation is also different. To this effect, suppose that the partition of the joint payoff from separation is given by U in figure 1 in case of unilateral divorce, while it is still given by C in the case of consensual divorce. Now a marriage yielding the utility pair P_3 would have efficiently survived under unilateral divorce. On the other hand, the result above implies that under fault divorce there exists an equilibrium in which the same marriage is inefficiently terminated. Furthermore, in such a case not only would Betty have preferred the marriage to go through, but she is worse off than if separation had been unilateral. In other words, she is forced to make some concession rather than being compensated. The intuition for this is that Anthony can refrain from cooperating in the marriage. Under such conditions Betty is willing to pay in order to go free and pursue her life.

In the inefficient consensual divorce equilibrium, the probability of marriage survival may be higher or lower than under unilateral divorce. This depends on the joint distribution of the individual gains from marriage which determines the probability mass to the north-east of U and C respectively.

The intuition that consensual divorce may actually damage rather than benefit the spouse that does *not* want to initiate the separation may appear surprising, yet it is easily understood once one realizes that the party that wants divorce may effectively hold up the other spouse by threatening not to cooperate in the marriage. This result also distinguishes the prediction of our model from all other models of marital separation that treat a cooperative marriage as an option. Those models predict that, under consensual divorce, it is always the spouse who wants to initiate divorce that compensates the other

partner. Our model predicts that instead the spouse who wants to divorce may be able to extract a payment from the other spouse.

Evidence supporting such mechanism is provided by cases of Jewish women who have been divorced in civil courts (or abandoned) by their husbands, but have not been given a religious divorce[4]. According to Jewish law only the husband can legally terminate the marriage by giving the wife a bill of divorce: a *get*. A wife has to accept such a document for the divorce to be valid. In this sense, Jewish divorce is consensual. Yet, the consequences of either spouse's not consenting to divorce are very different. A woman who has not obtained a *get* cannot have a relationship with another man without committing adultery and any child born out of a new relationship is considerate illegittimate and cannot marry another Jew. A husband who remarries without his former wife consenting to divorce is not guilty of adultery but of poligamy (a rabbinic not a Biblical prohibition). The children born from the union with a free Jewish woman are legitimate Jews. The fact that a husband divorces his wife in a civil court indicates that he is better off outside the relationship. If consensual divorce ensured that the spouse who did not initiate divorce were compensated, one would expect if any such husband to compensate his wife in order for her to accept a *get*. Yet, it is not uncommon for husbands to divorce their wives in civil court and refusing a *get* as a bargaining ploy to extract financial concessions or child. The Jewish Chronicle reports that some husbands demand sums ranging between £10,000 and £60,000 in return for a *get*. That this is more than a theoretical possibility is also confirmed by the 1983 amendment to the New York Domestic Relations Law which is often referred to as the "New York *Get* Law". Such amendment denies a plaintiff the right to civil divorce until s/he has taken all steps within his power to remove barrier's to the defendant's remarriage.

3 The Model

Anthony and Betty are married, and jointly enjoy their (commonly owned) assets, which we call 'the house'. Let the total market value of the house be equal to $h > 0$. We

[4]There are also cases of Jewish men not being able to convince their wives to consent to a religious divorce. The plight of "chained" wives (agunot) is much more common though.

assume that the value of each spouse from the (perpetual) joint consumption of the house is equal to $\frac{h}{2}$ (asymmetries between spouses in this respect are not crucial for our main argument). Once sold, the proceeds from the house are fully transferable between spouses. Anthony and Betty are discussing on a major decision (such as taking up a new job which involves a major relocation). Let us call this decision 'the investment'. If agreement can be obtained on the investment, the relationship generates *additional* utility for each, depending on the fondness they have for each other and on the specific utility from the investment[5]. Denote these utilities U_i^m for spouse $i \in \{A, B\}$, so that the total utility for a spouse from a happy marriage is $\frac{h}{2} + U_i^m$. Until agreement is reached on the investment, each spouse still benefits from the jointly owned house, but at the same time relinquishes the gratification of a happy marriage. Failure to reconcile marital disagreements can lead to divorce, and the consequent need to agree on a division of the assets. In case of divorce, each spouse i will enjoy some - non-monetizable - utility $u_i^s > 0$ from being single. So the maximum theoretical total utility for a spouse from divorce is $h + u_i^s$. However, divorce can only be obtained (and the house sold) after an agreement on the division of the proceeds of the asset.

Each partner can guarantee $\frac{h}{2}$ for himself or herself by always refusing to consent to divorce. On the other hand if divorce is agreed upon, each spouse's utility cannot be less than u_i^s. The set of feasible and individually rational agreements in case of divorce is then

$$D = \left\{ x \in \mathcal{R}^2 | \max\left\{\frac{h}{2}, u_i^s\right\} \leq x_i \leq h + u_i^s \text{ for } i = A, B \right\}$$

When this set is non-empty, the maximum utility from a feasible divorce agreement for spouse i which is compatible with individual rationality for the other spouse is

$$t_i^+ = \max\{x_i \in \mathcal{R} | (x_i, x_j) \in D \text{ for some } x_j\} = u_i^s + \frac{h}{2} + \min\left\{\frac{h}{2}, u_j^s\right\}$$

In fact, in case of separation spouse i can never extract from the partner j more than j's share of the house: but if j's utility of being single is lower than the value of half the

[5]Appendix 2 shows how the result in the paper goes through in the case in which the surplus is generated not if the parties reach an agreement on some major decision, but if they refrain from disrupting the marriage by asking for divorce.

house, then i can extract at most a share of the house equivalent to u_j^s (otherwise j would not rationally consent to divorce).

The set D is non-empty given that $u_i^s > 0$ for all i. We will restrict our attention to the case $h/2 > u_i^s$ for all i, which guarantees that the disagreement point is interior to the bargaining set[6]. Otherwise, essentially the same analysis would still go through, but there would be some corner solutions in the bargaining game. A possible bargaining set is depicted in Figure 2.

Although the maximum feasible utilities differ between the spouses (unless $u_i^s = u_j^s$), note that in the case under consideration - $h/2 > u_i^s$ - the individual rationality constraint equalises the maxima[7], with

$$t_i^+ = t^+ = \frac{h}{2} + u_i^s + u_j^s$$

To simplify notation it is convenient to treat the disagreement point $\left(\frac{h}{2}, \frac{h}{2}\right)$ as the origin. With this normalisation denote the total surplus from marriage $y = \left(U_A^m - \frac{h}{2}\right) + \left(U_B^m - \frac{h}{2}\right) = u_A^m + u_B^m$ and the total surplus in case of separation $z = t^+ - \frac{h}{2} = u_i^s + u_j^s$.

The relationship game can be described at follows.

[Figure 2 here]

There is a potentially unbounded number of periods indexed by $n = 0, 1, \ldots$ over which Anthony (A) and Betty (B) alternate in proposing either to stay married or to divorce. Whenever spouses disagree, their utility comes solely from the enjoyment of the commonly owned assets. In case the proposal to stay married is accepted, the game ends with each agent i obtaining the fixed amount u_i^m. Proposing divorce entails offering some share of the assets. The responder can either accept, ending the game; or reject. In this case play moves to the next period after a delay Δ. In the next period, the previous responder can either purpose to stay married or divorce, and so on. Perpetual

[6] As shown in Rubinstein (1982), even if this assumption does not hold, the alternating offer bargaining game has a unique subgame perfect equilibrium outcome. Manzini and Mariotti (2001) extend this result to the case with non-linear utility frontier.

[7] This is not the case in general. We illustrate one such example in Figure 3, where for one agent (Anthony) the utility from being single exceeds the utility from being unhappily married, i.e. $u_A^s > \frac{h}{2}$, while $u_B^s < \frac{h}{2}$. Then it is easy to see that $t_A^+ = \frac{h}{2} + u_A^s + u_B^s$, whereas for Betty $t_B^+ = h + u_B^s < t_A^+$.

disagreement (i.e. haggling over divorce) results in each spouse receiving half of the assets. The parties discount the future at the common instantaneous exponential rate r. Hence, agent i's utility from an agreement yielding x in round n is given by $u_i(x, n) = \delta^n x$, where $\delta = e^{-r\Delta}$. We assume that Betty starts first.

[Figure 3 here]

4 Results

In what follows we derive the equilibrium under consensual divorce. The equilibrium concept we shall rely upon is that of subgame perfect equilibrium (s.p.e.). We show that both marriage equilibria (i.e., equilibria where Anthony and Betty stay together happily) and divorce equilibria can obtain.

Divorce equilibria can be distinguished into two main categories, depending on how divorce arrangements are arrived at. In one case, the two spouses simply ignore the benefits of marriage in their divorce proceedings, and the surplus from separation is divided according to the standard Rubinstein shares. In this class of equilibria both agents always propose divorce. We call these *plain divorce* (**pd**) equilibria. These equilibria obviously occur when $y < z$, as noted in section 2**,** but more strikingly they can also obtain when $y > z$.

On the other hand, there are equilibria where one of the two spouses favours marriage over divorce. Here the party that stands to lose more from not being married is "compensated" in divorce: instead of getting the Rubinstein share, s/he gets the (discounted) value of the utility in marriage. We call equilibria in this class *compensating divorce* (**cd**) equilibria.

The marriage equilibria can also be distinguished along similar lines according to the equilibrium strategies that support the decision to remain married, i.e. *plain marriage* (**pm**) when both spouses prefer to propose marriage to divorce, and *bossy marriage* (**bm**) when the responder would propose to divorce if s/he got a chance (i.e. if s/he were the first proposer).

Below we formalise these results. The following proposition establishes conditions under which agreement on a divorce settlement is an equilibrium.

Proposition 1 (Divorce equilibria) *If and only if either*

(pd.i) $u_i^m \leq \frac{\delta}{1+\delta} z$ *and* $u_j^m \geq \frac{\delta}{1+\delta} z$, *or*

(pd.ii) $u_i^m, u_j^m \in \left[\frac{\delta}{1+\delta} z, \frac{1}{1+\delta} z\right]$, $i, j = A, B$, *or*

(pd.iii) $u_i^m \leq \frac{\delta}{1+\delta} z$ *for all* i,

then there exists an s.p.e. where Anthony and Betty agree immediately on a divorce settlement which yields $\frac{1}{1+\delta} z$ *to Betty and* $\frac{\delta}{1+\delta} z$ *to Anthony. Moreover, if and only if*

(cd) $z - \delta u_A^m > u_B^m$, $\delta\left(z - \delta u_A^m\right) < u_B^m$ *and* $u_A^m > \frac{1}{1+\delta} z$

then there exists an s.p.e. where Anthony and Betty agree immediately on a divorce settlement which yields $z - \delta u_A^m$ *to Betty and* δu_A^m *to Anthony.*

Proof: See appendix 1[8].

Proposition 1 characterizes two types of divorce equilibria. In equilibria of the first type (**pd**) both parties propose and accept to share the joint payoff from separation according to Rubinstein's partition. Equilibria of the second type (**cd**) are supported by the first proposer - Betty - always proposing to divorce and Anthony always proposing to cooperate in the marriage. Since delay is costly, when respondng Betty is better off accepting to cooperate than rejecting and proposing in the next round. Hence, when first proposing she has to offer Anthony the discounted utility from a cooperative marriage that he would obtain by rejecting the current offer.

The next proposition characterizes the equilibria where Anthony and Betty cooperate within the marriage.

Proposition 2 (Marriage equilibria) *If and only if either*

(pm) $z - \delta u_i^m < u_j^m$, $i, j = A, B$, *or*

(bm) $z - \delta u_B^m > u_A^m$, $\delta\left(z - \delta u_B^m\right) < u_A^m$ *and* $u_B^m > \frac{1}{1+\delta} z$

[8]Note that the conditions for the **(cd)** equilibrium require that δ be 'sufficiently' small. This point is discussed in the appendix.

there exists an s.p.e. where Anthony and Betty agree immediately on staying married.

Proof: See appendix 1[9].

As for proposition 1, there are two classes of marriage equilibria. In equilibria of the type (**pm**) both parties propose and accept cooperation within the marriage. Equilibria of the second type (**bm**), instead, are characterized by the first proposer - Betty - offering to cooperate within the marriage and Anthony accepting. Yet, if Anthony were to propose he would offer Betty the discounted value of her utility from a cooperative marriage in exchange for her agreement to divorce.

As we show in appendix 1, propositions 1 and 2 fully carachterise *all stationary* subgame perfect equilibria of the game. The various parameter configurations corresponding to the equilibria of propositions 1 and 2 are depicted in Figure 4. The axes measure agents' payoffs, both in case of divorce and if staying married. The line **zz** is the locus of possible partitions of the divorce surplus, z. On the same quadrant we can also represent various points corresponding to the marriage surplus u^m, with coordinates u_A^m and u_B^m. The other lines are needed to determine the various configurations of parameters that satisfy the various (inequality) conditions introduced in the statement of Propositions 1 and 2. The position of u^m determines the equilibrium outcome. So for instance if point u^m were to fall into the light shaded region delimited by $u_i^m < \frac{\delta}{1+\delta} z$ for $i = A, B$, from proposition 1 we see that **pd.iii** would be the corresponding stationary equilibrium. It is easy to verify that in cases where $u_A^m + u_B^m = y < z$, only plain divorce equilibria can obtain. The crucial and novel result, though, is that there is a whole range of parameter configurations - when the utility pair from the marriage falls in the shaded area to the right of the **zz** line - in which *separation is an equilibrium despite being inefficient.* Regarding marriage equilibria, not surprisingly given free transferability in case of separation, there are no marriage equilibria when separation is efficient; i.e. when the utility pair associated with a cooperative marriage lies to the right of the line **zz** ($u_A^m + u_B^m = y < z$). Figure 4 also shows that there is a range of parameter values such that both divorce and marriage equilibria coexist, despite the fact that continuation of the marriage maximizes

[9]Note that the conditions for the **(bm)** equilibrium require that δ be 'sufficiently' small. This point is discussed in the appendix.

joint wealth. This is the case when the utility pair from a cooperative marriage falls in the overlap areas delimited by the thick dashed lines.

It is important to underline that transaction costs - namely the value of the discount factor - are crucial in determining whether or not the equilibrium configuration **bm** and the symmetric **cd** may arise. Recall that in these equilibria one agent prefers marriage over divorce in subgames in which she is the proposer, whereas the converse holds for the other spouse. In order for a divorce settlement to be accepted, there must be enough resources such that for the responder divorce is at least as attractive as marriage (which otherwise the dissatisfied spouse could propose in the next period), while at the same the party who prefers divorce finds it still worthwhile. Since such equilibria exist only when $y > z$, it must be that the cost of haggling over a divorce settlement (embodied in the discount factor) is high enough to make up for this shortfall in resources. Consequently, if transaction costs are sufficiently low (i.e. the discount factor is sufficiently high), marriage becomes irresistibly attractive for at least one of the two spouses, so that for this party it is never optimal to accept divorce. In terms of Figure 4, the greater δ, the closer point **b** moves towards point **a**, as the graphs of $u_i^m = z - \delta u_j^m$ pivot inwards toward the **zz** line while at the same time the graphs of $u_i^m = \delta\left(z - \delta u_j^m\right)$ rotate clockwise, also closing towards **zz**. In the limit as $\delta \to 1$ the two points (and the four graphs) collapse onto the line **zz**, and there is no point u^m that can satisfy all the optimality requirements for these equilibria.

[**Figure 4 here**]

Both the extent to which marriage and divorce equilibria coexist and inefficient separation is a possible equilibrium depend on the size of transaction costs captured by the discount factor. The mechanism at play becomes more evident in the benchmark limit case in which δ converges to one. For δ arbitrarily close to one, it follows from proposition 2 that it is an equilibrium for the parties to agree to cooperate in the marriage if and only if $y > z$. In particular, in the limit $y > z$ implies that **pm** is always an equilibrium. On the other hand, proposition 1 implies that unless both parties are better off in a cooperative marriage - the pair u_A^m, u_B^m lies to the north east of point C in figure **??** - the outcome **pd.i** in which the spouses agree to separate and share joint utility according to

the Rubinsteinian shares is also an equilibrium. As discussed in section 2, unless both spouses are better off cooperating in the marriage, both efficient separation and inefficient continuation are possible equilibria when $y > z$ and divorce has to be consensual.

As anticipated in section 2, if the ruling social convention implies that the inefficient equilibrium prevails separation takes place whenever one spouse prefers it to a cooperative marriage. In such equilibrium the outcome is identical to that under unilateral divorce. It is different only if under unilateral divorce the parties shares of the joint payoff from separation differ from those under consensual divorce.

As is standard in this type of literature[10], the coexistence of two subgame perfect equilibria guarantees the existence of a continuum of equilibrium outcomes, all involving divorce. These are completely characterized in the following proposition.

Proposition 3 *Let $y > z$, and assume $u_A^m \in \left(\delta\left(z - \delta u_B^m\right), \frac{\delta}{1+\delta} z\right)$ and $u_B^m \geq \frac{1}{1+\delta} z$. Then all divorce settlements with $x^* \in \left[\frac{1}{1+\delta} z, z - u_A^m\right]$ can be supported in a divorce equilibrium with immediate agreement on $(x^*, z - x^*)$.*

Proof: See appendix 1.

Note that one can construct multiple equilibria, all involving divorce, even when δ converges to one and the **cd** and **bm** type of equilibria are not defined. As specified below, these rely on the same equilibrium strategies as those supporting the equilibrium in proposition 3.

Proposition 4 *Let $y > z$, and assume $u_A^m \in \left(z - \delta u_B^m, \frac{\delta}{1+\delta} z\right)$ and $u_B^m \geq \frac{1}{1+\delta} z$. Then all divorce settlement with $x^* \in \left[\frac{1}{1+\delta} z, z - u_A^m\right]$ can be supported in a divorce equilibrium with immediate agreement on $(x^*, z - x^*)$.*

Proof: See appendix.

Remark 5 *Other divorce equilibria corresponding to the symmetric parameter configurations of propositions 3 and 4 can be derived inverting all the subscripts for Anthony and Betty's payoffs if happily married.*

[10] See e.g. Muthoo (1999).

5 Comparative statics

It is interesting to investigate the effects that alternative law provisions, changes in the value of the assets and in the utility from being single may have on the configuration of equilibria. These changes could take place exogenously (e.g. a change in preferences), or be the effect of a spouse's "investment" decision. For instance, Anthony could invest in plastic surgery, and thus become more attractive to the opposite sex if single, thereby pushing his utility outside marriage upwards. The following sections analyse these effects more in detail.

5.1 Being single

The consequence of changes in u_i^s for a spouse is simply to move the bargaining set, *not* the disagreement point, which depends on the utility from the assets, and is therefore fixed. We consider changes in u_i^s such that u_i^s remains below $\frac{h}{2}$, as in section 4. Specifically, an *increase* in u_i^s for a spouse "pushes" the bargaining set upwards or to the right, so that the effect is simply to increase the overall surplus available in the case of separation, $u_A^s + u_B^s$. Consequently, assuming the type of equilibrium prevailing does not change, it is easy to see that there are three possible comparative statics effects:

- In marriage equilibria equilibrium payoffs are unaltered.
- In plain divorce equilibria, both spouses benefit from the increase in the surplus from separation.
- In the bossy divorce equilibrium the first mover is able to appropriate the entire increase in surplus, while the responder's payoff remains unchanged (since u_i^m is unchanged).

Common sense would suggest that investing to improve one's opportunities when single should strengthen the bargaining position in marriage. To the contrary, our model shows that matters are more complex. Indeed, in the equilibrium regime where marriage is not the outcome, the spouse whose utility as single has *not* increased will be able to appropriate at least some of the enhanced opportunities of the partner: such enhanced opportunities simply add to the overall stake which is being negotiated.

5.2 Assets

Consider now the effect of a change in the value of the house. In this case both the disagreement point and the size of the bargaining set (and of the individually rational subset) change. For instance, in case of a reduction of the house value the set of individually rational allocations would shrink: the possibility of transfers has diminished. An interesting implication of this fact is that coeteris paribus one would expect to observe fewer consensual separations in households which are poorer in term of assets, relative to the wealthier ones.

If the assets reduce enough in value (shifting the disagreement point to the south west of the point (u_B^s, u_A^s)), all the allocations in the bargaining set become individually rational. In this case if plain divorce equilibria survive, they may imply an asymmetric division of the surplus from separation $(u_B^s + u_A^s)$ - in which one of the partners gets more than half of the surplus - even for δ tending to one. On the contrary, in the case we have considered in section 4 the disagreement point and the bargaining set are symmetric, so that in the limit as δ approaches one, the spouses share the surplus from separation equally.

6 Concluding Remarks

Our results can be understood in the light of the property-rights theory of the firm. Under unilateral divorce, each partner has residual control rights on his/her participation in the marriage. While it is uncontroversial that the option to divorce can be unilaterally exercised under at-will divorce, it is not the case that under consensual divorce the spouse that wants the marriage to continue has control over the other spouse's cooperation. In other words, he or she has the power to veto marriage termination but not the right to a cooperative marriage (which is obviously non-contractible). Residual control rights are left unallocated under consensual divorce. So the outcome is determined by bargaining. While this has no effect on the separation decision if utility is transferable at the same rate both inside the marriage and in case of separation, it has fundamental implications in all other cases. The marriage survives if it strongly Pareto dominates the agreement on divorce. If this is not the case, the two equilibria in which the marriage efficiently

survives and inefficiently terminates are not Pareto ranked. The spouses are, in a sense, playing a battle of the sexes. One can argue that which of the two equilibria prevails is a matter of social convention.

The above suggests that our framework can be extended to the theory of investment in general productive relationships - we leave this issue open for further research.

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Appendix 1: Proofs

Proof of propositions 1 and 2.

Supporting strategies are described in Table 1, where we adopt the convention that the first entry of a given partition refers to the share of the proposing agent. Checking that each profile is an s.p.e. is straightforward thus omitted.

[**Table 1 here**]

The equilibrium partitions in Table 1 can be derived as follows. Let P_i^j, $i, j = A, B$ be the equilibrium payoff for player i in subgames where player j either offers to stay married or proposes a divorce settlement (subgames of type G^j). Furthermore, let r_i^j, $i, j = A, B$, denote the equilibrium payoff to player i in subgames starting with the decision of player j whether to accept staying married or propose a divorce settlement in the next period (subgames of type H^j). Assume that there is immediate agreement. Then the following system of equation must be satisfied in a stationary equilibrium:

$$P_B^B = \max\left\{z - \delta P_A^A, r_B^A\right\} \tag{1}$$

$$r_B^B = \max\left\{\delta P_B^B, u_B^m\right\} \tag{2}$$

$$P_A^A = \max\left\{z - \delta P_B^B, r_A^B\right\} \tag{3}$$

$$r_A^A = \max\left\{\delta P_A^A, u_A^m\right\} \tag{4}$$

The first two equations refer to subgames of type G^B and H^B, respectively, whereas the last two equations refer to subgames of type $G^A$and H^A, respectively. Depending on parameter values, the unique solution to the above system defines the equilibrium outcomes of Table 1. The solution depends on the direction of each of four sets of inequalities:

$$z - \delta P_A^A \gtreqless r_B^A \tag{a}$$

$$z - \delta P_B^B \gtreqless r_A^B \tag{b}$$

$$\delta P_A^A \gtreqless u_A^m \tag{c}$$

$$\delta P_B^B \gtreqless u_B^m \tag{d}$$

In what follows we use the suffix .1 whenever the L.H.S. is greater than the R.H.S., and the suffix .2 when the opposite is true. So for instance b.2 is a shorthand for $z - \delta P_B^B < r_A^B$. This generates sixteen possible sets of inequalities, several of which generate inadmissible parameter values, leaving only seven valid inequalities, each corresponding to one of the stationary equilibria described in propositions 1 and 2, as we show below. We start by distinguishing four main cases as obtained by the various combinations of the inequalities sub a and b. The direction of the two remaining inequalities determine four possible subcases for each of the main cases.

As a preliminary, note that $z - \delta P_j^j > r_i^j$ implies that in subgames of type G^i agent i prefers to propose the equilibrium divorce settlement (which yields $z - \delta P_j^j$) rather than propose marriage (which yields r_i^j). The opposite is true if the direction of the inequality is reversed. Similarly, $\delta P_i^i > u_i^m$ implies that in subgames of type H^i agent i prefers to accept marriage rather than obtain the continuation payoff in a subgame of type G^i in the next round. We can now analyse all possible admissible configurations of the parameters.

Case 1

$$z - \delta P_A^A > r_B^A \text{ and } z - \delta P_B^B > r_A^B \tag{5}$$

In this case in subgames of type G^i both agents achieve a higher payoff by proposing a divorce settlement rather than by pursuing marriage. Consequently equations 1 and 3 collapse to those characterising a standard bilateral monopoly bargaining over a surplus of size z, which results in the equilibrium partition which gives $\frac{1}{1+\delta}z$ to the proposer and $\frac{\delta}{1+\delta}z$ to the responder, so that $P_B^B = P_B^B = \frac{1}{1+\delta}z$. In this case the equilibrium outcome is therefore always going to be of the "plain divorce" type. The direction of inequalities sub c and d is going to determine the equilibrium strategies off the equilibrium path, as

shown below.

Subcase 1.1

$$\delta P_A^A > u_A^m \text{ and } \delta P_B^B > u_B^m \tag{6}$$

In subgames of type H^i both agents prefer to get the proposer's payoff in the next round rather than ending the game with the marriage payoff. This implies that the payoff in subgames H^i is δP_i^i to agent i (who rejects marriage and proposes the equilibrium plain divorce settlement in subgame G^i in the following round); and $\delta P_j^i = \delta\left(z - P_i^i\right)$ to player j, so that $r_j^i = \delta\left(z - P_i^i\right)$. So, the equilibrium strategy profile is the one described under **(pd.iii)** in table 1.

Subcase 1.2

$$\delta P_A^A < u_A^m \text{ and } \delta P_B^B > u_B^m \tag{7}$$

Now in subgames H^i it is only one of the agents (Betty) who prefers divorce to marriage in subgames of type H^B. In this subgames it is optimal for her to reject marriage and propose the bilateral monopoly divorce settlement, whereas iin subgames of type H^A Anthony prefers accepting marriage to his continuation payoff in the following round. This readily implies that $r_B^A = u_B^m$: if Betty where to propose marriage, triggering a subgame of type H^A, Anthony would accept (obtaining a payoff $r_A^A = u_A^m$). On the other hand, subgames of type H^B are as in subcase 1.1 above. This tallies with the strategy profile **(pd.i)** in Table 1 with $i = B$ and $j = A$.

Subcase 1.3

$$\delta P_A^A > u_A^m \text{ and } \delta P_B^B < u_B^m \tag{8}$$

This is symmetric to subcase 1.2 above, obviously with the strategies for Anthony and Betty reversed. Equilibrium strategies corresponds to those for equilibrium **(pd.i)** in Table 1 with $i = A$ and $j = B$.

Subcase 1.4

$$\delta P_A^A < u_A^m \text{ and } \delta P_B^B < u_B^m \tag{9}$$

In subgames of type H^i both agents prefer marriage to the continuation payoff in the following round (while anyway in subgames of type G^i it is still the case that both agents prefer to propose the equilibrium divorce settlement - immediately - rather than propose marriage). This explains the optimality of the equilibrium strategy profile **(pd.ii)** in Table 1.

Case 2

$$z - \delta P_A^A < r_B^A \text{ and } z - \delta P_B^B < r_A^B \tag{10}$$

This is the opposite situation as that described in case 1. The inequalities in 10 imply that in subgames of type G^i both agents prefer to propose marriage to a divorce settlement.

Subcase 2.1

$$\delta P_A^A > u_A^m \text{ and } \delta P_B^B > u_B^m \tag{11}$$

This case implies perpetual disagreement: because of inequalities in 10 agents always propose marriage in subgames G^i, but because of the inequalities in11 agents never accept marriage in subgames of type H^i. Note however that this cannot be an equilibrium. For instance, any of the two agents could profitably deviate in a subgame of type H^i and accept marriage, obtaining a positive payoff rather than the null payoff that perpetual disagreement entails.

Subcase 2.2

$$\delta P_A^A < u_A^m \text{ and } \delta P_B^B > u_B^m \tag{12}$$

The first inequality in 12 implies that Anthony would accept marriage in subgames of type

H^A, so that from equation 4 $r_A^A = u_A^m$ and $r_B^A = u_B^m$. Recall that from the first inequality in 10 Betty proposes marriage in subgames G^B. Since, as we just saw, Anthony would accept that, then it follows that $P_B^B = u_B^m$. But this is incompatible with the second inequality in 12. Thus, this scenario is impossible.

Subcase 2.3

$$\delta P_A^A > u_A^m \text{ and } \delta P_B^B < u_B^m \tag{13}$$

This case is symmetric to subcase 2.2 above, thus not admissible.

Subcase 2.4

$$\delta P_A^A < u_A^m \text{ and } \delta P_B^B < u_B^m \tag{14}$$

These two inequalities imply that $P_B^B = r_B^B = u_B^m$ and $P_A^A = r_A^A = u_A^m$ (from equations 2 and 4), so that $r_A^B = u_A^m$ and $r_B^A = u_B^m$. This corresponds to the strategy profile **(pm)** in Table 1.

Case 3

$$z - \delta P_A^A > r_B^A \text{ and } z - \delta P_B^B < r_A^B \tag{15}$$

The above inequalities imply that while Anthony prefers marriage to proposing a divorce settlement to Betty in subgames of type G^A, the opposite is true for Betty in subgames of type G^B. Note that substitution in equation 3 yields $P_A^A = r_A^B$.

Subcase 3.1

$$\delta P_A^A > u_A^m \text{ and } \delta P_B^B > u_B^m \tag{16}$$

Equations 1-4 reduce to

$$
\begin{aligned}
P_B^B &= z - \delta P_A^A \\
r_B^B &= \delta P_B^B \\
P_A^A &= r_A^B \\
r_A^A &= \delta P_A^A
\end{aligned}
$$

which has unique solution $P_A^A = r_A^B$, $r_B^B = \delta\left(z - \delta r_A^B\right)$, $P_B^B = z - \delta r_A^B$ and $r_A^A = \delta r_A^B$. Substituting back into the four inequalities defining this case yields:

$$
\begin{aligned}
&\mathbf{a.1}: \ z - \delta r_A^B > r_B^A \\
&\mathbf{b.2}: \ z - \delta\left(z - \delta r_A^B\right) < r_A^B \Longleftrightarrow r_A^B > \frac{1}{1+\delta} z \\
&\mathbf{c.1}: \ \delta r_A^B > u_A^m \\
&\mathbf{d.1}: \ \delta\left(z - \delta r_A^B\right) > u_B^m
\end{aligned}
$$

So in this hypothetical scenario along the (stationary) equilibrium path Betty would propose the divorce settlement δr_A^B. Off the equilibrium path, if rejecting Anthony would then propose marriage (because of inequality **b.2** in 15) which Betty would reject (because of inequality $d.1$ in 16) to make another divorce settlement proposal to Anthony of δr_A^B in the following round. This means that Anthony's equilibrium payoff in subgames of type H^B, r_A^B, would have to be equal to $\delta\left(\delta r_A^B\right)$, or $r_A^B = \delta^2 r_A^B$, which is possible only if $r_A^B = 0$, which contradicts condition **b.2** above.

Subcase 3.2

$$\delta P_A^A < u_A^m \text{ and } \delta P_B^B > u_B^m \tag{17}$$

These inequalities imply that $r_A^A = u_A^m$,with $r_B^A = u_B^m$: in words, if Betty were to propose

marriage, Anthony would accept. System 1-4 reduces to

$$
\begin{aligned}
P_B^B &= z - \delta P_A^A \\
r_B^B &= \delta P_B^B \\
P_A^A &= r_A^B \\
r_A^A &= u_A^m
\end{aligned}
$$

which is solved by $P_A^A = r_A^B$, $r_B^B = \delta\left(z - \delta r_A^B\right)$, $P_B^B = z - \delta r_A^B$ and $r_A^A = u_A^m$.. Substituting back into the four inequalities defining this case yields:

$$
\begin{aligned}
\textbf{a.1}: &\ z - \delta r_A^B > u_B^m \\
\textbf{b.2}: &\ z - \delta\left(z - \delta r_A^B\right) < r_A^B \Longleftrightarrow r_A^B > \frac{1}{1+\delta} z \\
\textbf{c.2}: &\ \delta r_A^B < u_A^m \\
\textbf{d.1}: &\ \delta\left(z - \delta r_A^B\right) > u_B^m
\end{aligned}
$$

A line of reasoning analogous to the one for subcase 3.2 shows that in such an equilibrium in subgames H^B Anthony's payoff would have to be $r_A^B = 0$, which contradicts condition **b.2**.

Subcase 3.3

$$\delta P_A^A > u_A^m \text{ and } \delta P_B^B < u_B^m \tag{18}$$

This case is symmetric to subcase 3.2 above, only with the roles of Anthony and Betty reversed, and admits no stationary equilibrium.

Subcase 3.4

$$\delta P_A^A < u_A^m \text{ and } \delta P_B^B < u_B^m \tag{19}$$

In this case in subgames of type H^i both agents favour marriage over triggering a subgame of type G^i in the next round. Consequently $r_B^B = r_B^A = u_B^m$ and $r_A^A = r_A^B = u_A^m = P_A^A$,

which also implies that $P_B^B = z - \delta u_A^m$. So, along the equilibrium path Betty proposes a divorce settlement, which is accepted by Anthony. This scenario corresponds to the **cd** equilibrium in Table 1. Note that substituting the equilibrium values into inequality **a.1**, which defines case 3, yields $z - \delta u_A^m > u_B^m$, or $z > \delta u_A^m + u_B^m$. As δ increase the right hand side of this expression approaches y, so that in order for **a.1** to be compatible with our requirement that $u_A^m + u_B^m = y > z$ it must be that δ is sufficiently small.

Case 4

$$z - \delta P_A^A < r_B^A \text{ and } z - \delta P_B^B > r_A^B \tag{20}$$

This is symmetric to case 3 above, so we omit the details. Only note that now, though the equilibrium strategies are the same as described in subcase 3.4 above after inverting the roles of Anthony and Betty, the equilibrium outcome is different, since now Betty, the first mover, favours marriage over divorce in subgames of type G^B. Consequently along the equilibrium path Betty proposes marriage, which is accepted. This corresponds to strategy profile **(bm)** in Table 1.

Showing that no delayed stationary equilibria can exist is routine, thus omitted[11]. ■

Proof of proposition 3.

Supporting strategies are as follows: Along the equilibrium path Betty proposes the divorce settlement of proposition 3, which Anthony accepts. Both agents punish deviations by reverting to the "worst" equilibrium for the deviator, that is strategies supporting equilibrium **bd** if Anthony deviates, and strategies supporting **pd.i** if Betty deviates. Checking that these strategies are an equilibrium is straightforward, thus omitted. We just sketch what deters deviations on the equilibrium path. Consider Betty first. If she put forward a different agreement from the equilibrium one, Anthony, given his strategy, would reject and counteroffer the plain divorce equilibrium partition, which Betty would accept, obtaining a payoff of $\frac{\delta}{1+\delta}z$ in the following round, which at the time of the deviation is worth $\frac{\delta^2}{1+\delta}z < \frac{1}{1+\delta}z \leq x^*$. Turning now to Anthony, if he rejected Betty's

[11]See for instance chapter 3 in Muthoo (1999).

equilibiurm offer, in the next round he could either propose to stay married, or make a divorce settlement. In the former case, Betty would accept, yielding Anthony a payoff of u_A^m, worth $\delta u_A^m < u_A^m \leq z - x^*$, so that such deviations would not be profitable. If instead Anthony were to propose a divorce settlement, it would have to be the one corresponding to the strategies for the **m.i** equilibrium, yielding at the time of the deviation a payoff in present discounted value equal to $\delta\left(z - \delta u_B^m\right) < u_A^m \leq z - x^*$. ■

Proof of proposition 4.

Supporting strategies are as those of proposition 3. The only change is that now the punishment triggered by Anthony's deviation is that play reverts to the strategy profile supporting the equilibrium **m.ii**. Reasoning analogous to that in the proof of proposition 3 shows that this is indeed an equilibrium, so we omit the details. ■

Appendix 2: an extension with bargaining over flows.

Our basic model can be modified to capture the situation in which the surplus from the marriage is always available (rather than being dependent on some investment-type decision such as having a child) unless one party destroys it.

In each bargaining round either party may propose to divorce or may simply do nothing. Doing nothing allows the couple to enjoy the gains from the marriage for the current round and does not end the game. The instantaneous flow payoffs from an undisrupted marriage are time-invariant and given by u_A^m and u_B^m (these are gains net of the enjoyment of the common house). The parties discount the future at the common exponential rate r. The expected present value of a stock Δ into the future is $\delta = e^{-r\Delta}$ and the expected present value of a unit flow payoff over a period of length Δ is given by $(1-\delta)/r = \int_t^{t+\Delta} e^{-r(s-t)ds}$. If the total surplus from separation z has the dimension of a stock then continuation is efficient if and only if

$$\frac{u_A^m + u_B^m}{r} \geq z. \tag{21}$$

If either party proposes to divorce in any round, the couple foregoes the gains from (a

blissful) marriage for that round[12]. If divorce is proposed and accepted the game ends, otherwise it moves to a new round.

This game has still two multiple (stationary) subgame perfect equilibria if continuation is efficient.

Proposition 6 *Let* $z \leq \min\left\{\frac{u_A^m+\delta u_B^m}{r}, \frac{\delta u_A^m+u_B^m}{r}\right\}$*, which implies that continuation is efficient. Then it is a stationary SPE for the marriage to continue.*

Proof. Supporting strategies:

1. Both parties do nothing whenever it is their turn to propose.

2. Agent i accepts a share $\geq \delta u_i^m/r$ and rejects otherwise.

One-stage deviation at point 1. Given the equilibrium strategies, it is not optimal to make a separation proposal that is rejected, since this just destroys the gains from a blissful marriage for the current round. Consider then a deviation in which party i makes an acceptable divorce proposal. In such a case he/she obtains $z - \delta u_{-i}^m/r$. If he/she plays the equilibrium strategy, he/she gets instead u_i^m/r.

So the above strategy is optimal whenever it is $z - \delta u_{-i}^m/r \leq u_i^m/r$, that is whenever continuation is efficient.

It is trivial to show that the part of the strategy characterized at point 2 satisfies the one-stage deviation condition. ■

Proposition 7 *If* $z \geq u_i^m/r$ *with* $i = A, B$*, it is a SPE for the marriage to end immediately with the joint payoff from separation being split according to the Rubinsteinian shares.*

Proof. Supporting strategies:

1. Both parties propose to separate with the proposer i offering the responder $-i$ a share $\pi_{-i}^* = \delta z/(1+\delta)$.

[12]The crucial difference between this set up and Fernandez and Glazer (1991) is that here a proposal to separate destroys the gains from cooperation for the current round. In Fernandez and Glazer the parties can still trade at the existing contract in the current round even if either party has proposed to renegotiate the contract. Our set up seems to better describe a marriage situation.

2. When responding agent $-i$ accepts separation if offered $\pi_{-i} \geq \pi^*_{-i}$ and rejects otherwise.

Optimality of the part of the strategy described in point 2 follows from Rubinstein (1982). We just need to prove the optimality of proposing to separate rather than doing nothing. The Rubinstein's equilibrium payoff to the proposer is $z/(1+\delta)$. The payoff associated with a one-stage deviation would be $(1-\delta)\, u_i^m/r + \delta^2 z/(1+\delta)$. The first term is the payoff from an undisrupted marriage over the interval Δ. The second addendum is the present value of the payoff from being the responder in the following round. Proposing divorce is then optimal if $z \geq u_i^m/r \quad i = A, B$. ■

The above proposition implies that it is possible for the Rubinstein outcome to be an equilibrium even when it is Pareto dominated by the marriage outcome, viz. when it is $z \geq u_i^m/r \geq z/(1+\delta)$ for all i and $z \geq u_{-i}^m/r \geq \delta z/(1+\delta)$.

Proposition 8 *If $z \in [u_i^m/r + \delta u_{-i}^m/(1+\delta)\, r, u_{-i}^m/r]$, it is a SPE for the marriage to end when agent i proposes for the first time. In such equilibrium the parties enjoy the gains from the marriage until agent i proposes for the first time. On separation agent $-i$ receives $\pi^*_{-i} = \delta u_{-i}^m/(1+\delta)\, r$ and agent i receives $z - \pi^*_{-i}$.*

Proof. Supporting strategies:

1. Agent i always proposes to separate offering $-i$ a payoff $\pi^*_{-i} = \delta u_{-i}^m/(1+\delta)\, r$.

2. Agent i accepts separation if it yields $\pi_i \geq \delta\left(z - \delta u_{-i}^m/(1+\delta)\, r\right)$

3. Agent $-i$ does nothing when it is his turn to propose and accepts a separation offer π_{-i} if $\pi_{-i} \geq \pi^*_{-i}$ and rejects otherwise.

Optimality for $-i$ of accepting i's proposal of π^*_{-i}. If $-i$ accepts he obtains $\pi^*_{-i} = \delta u_{-i}^m/(1+\delta)\, r$. His payoff in case of a one-stage deviation is $\delta[(1-\delta)\, u_{-i}^m/r + \delta\pi^*_{-i}] = \delta[(1-\delta)\, u_{-i}^m/r + \delta^2 u_{-i}^m/(1+\delta)\, r] = \delta u_{-i}^m/(1+\delta)\, r$. This is the present value of being the responder in the next round and obtaining the payoff from an undisrupted marriage plus the present value of accepting divorce in the following round. Acceptance is optimal if $\pi_{-i} \geq \delta u_{-i}^m/(1+\delta)\, r$.

Optimality for $-i$ of doing nothing when proposing. Agent $-i$'s payoff from doing nothing is $(1-\delta)\, u_{-i}^{m}/r + \delta\pi_{-i}^{*}$. His payoff from a one-stage deviation when proposing would be $z - \delta\left(z - \pi_{-i}^{*}\right)$. Doing nothing is optimal if $u_{-i}^{m}/r \geq z$.

Optimality for i of proposing to make a separation offer of π_{-i}^{*}. If i makes such a proposal he obtains $z - \pi_{-i}^{*}$. Alternatively, his payoff from a one-stage deviation would be $\pi_i^d = (1+\delta)(1-\delta)\, u_i^m/r + \delta^2\left(z - \pi_{-i}^{*}\right)$. This corresponds to receiving the payoff from an undisrupted marriage in the next two rounds and divorcing two rounds away. Offering π_{-i}^{*} is optimal if it is $z - \delta u_{-i}^{m}/(1+\delta)\, r \geq u_i^m/r$. Also, it is not optimal for agent i to make a proposal which yields agent $-i$ less than π_{-i}^{*}: in this case agent $-i$ would reject, so that the payoff to deviant player i would be $\delta(1-\delta)\, u_i^m/r + \delta^2\left(z - \pi_{-i}^{*}\right) \leq \pi_i^d$, so that this deviation is not profitable.

Finally, it is optimal for agent i to accept separation only if it yields a payoff of at least $\delta\left(z - \delta u_{-i}^{m}/(1+\delta)\, r\right)$. In fact, by rejecting he could obtain his equilibrium payoff of $z - \pi_{-i}^{*}$ in the following round, or $\delta\left(z - \delta u_{-i}^{m}/(1+\delta)\, r\right)$. ■

This type of equilibria corresponds to the **cd** equilibria in the main text. Note that they do not disappear even if $\Delta \to 0$.

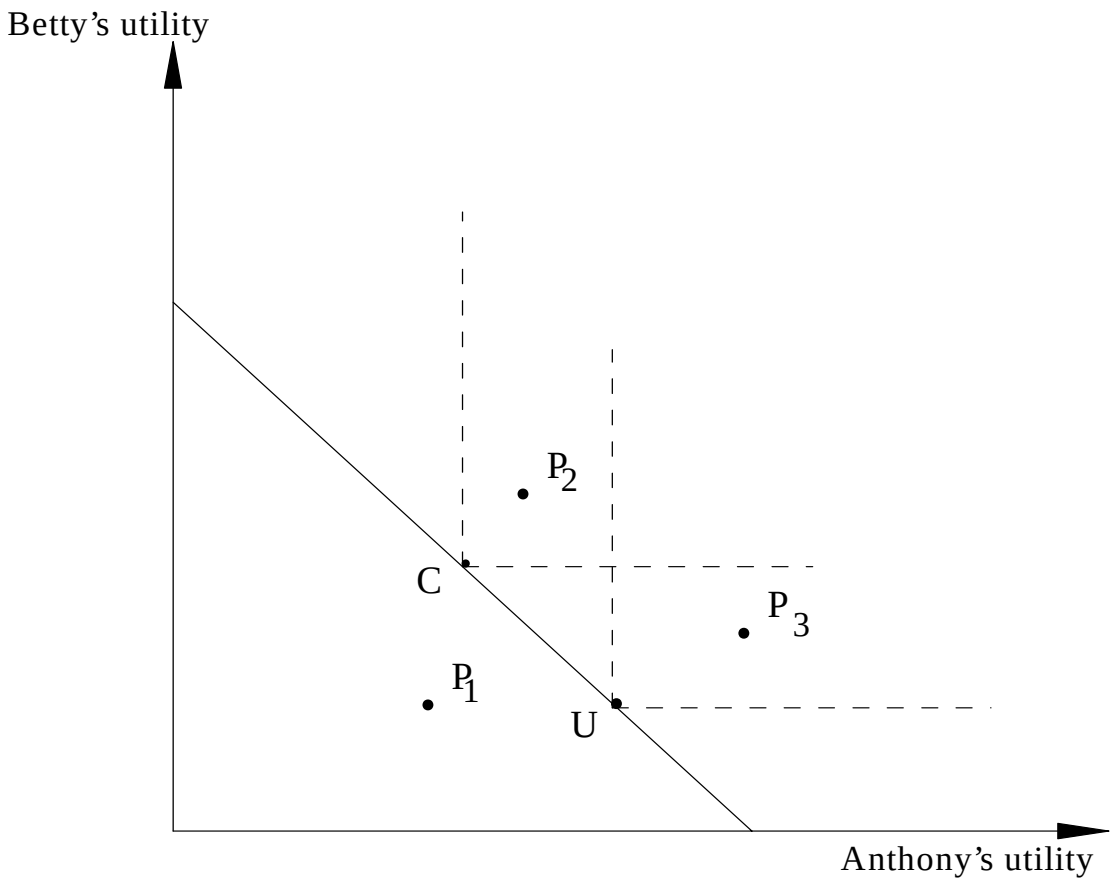


Figure 1: A simple example

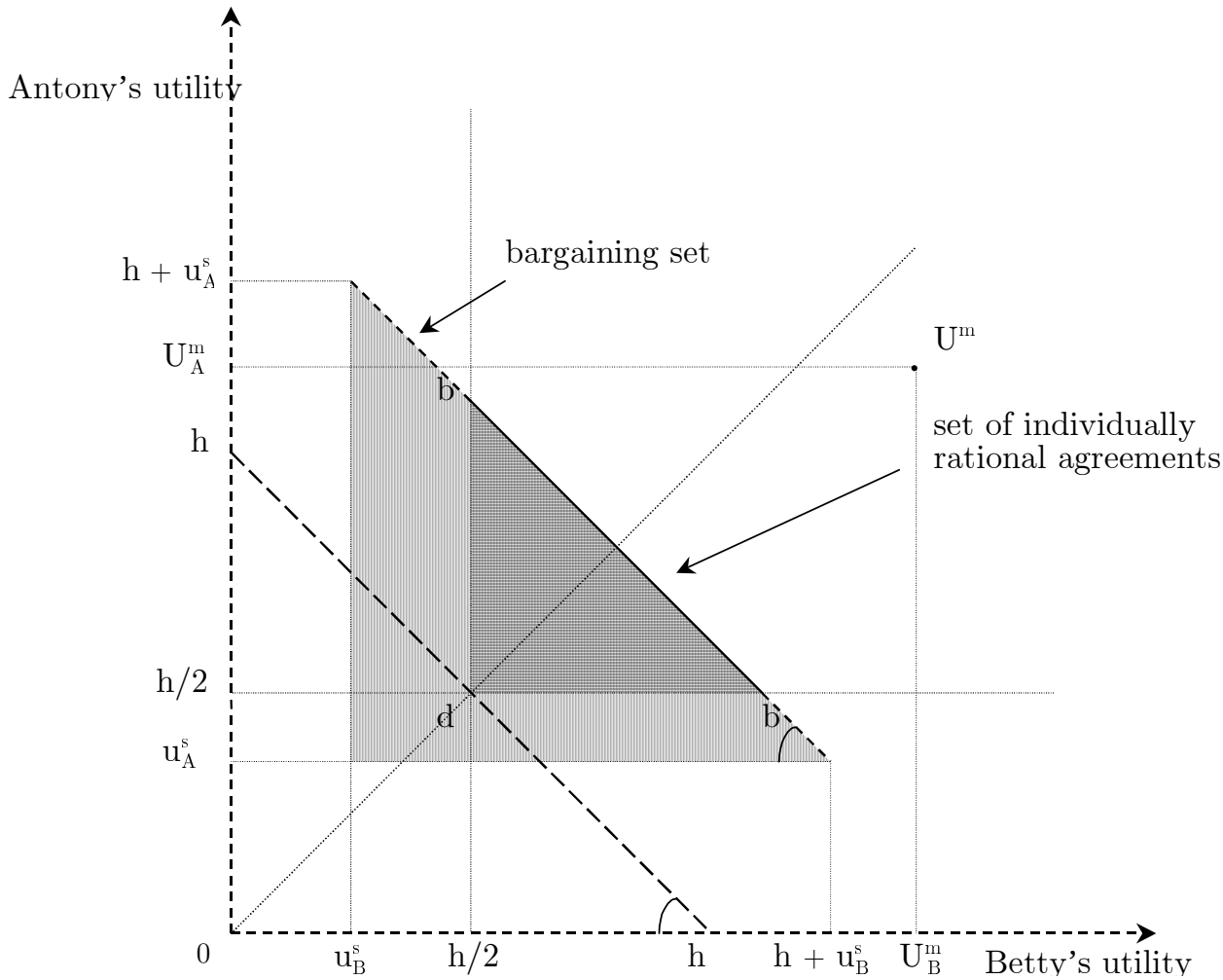


Figure 2: The bargaining set in the marriage game

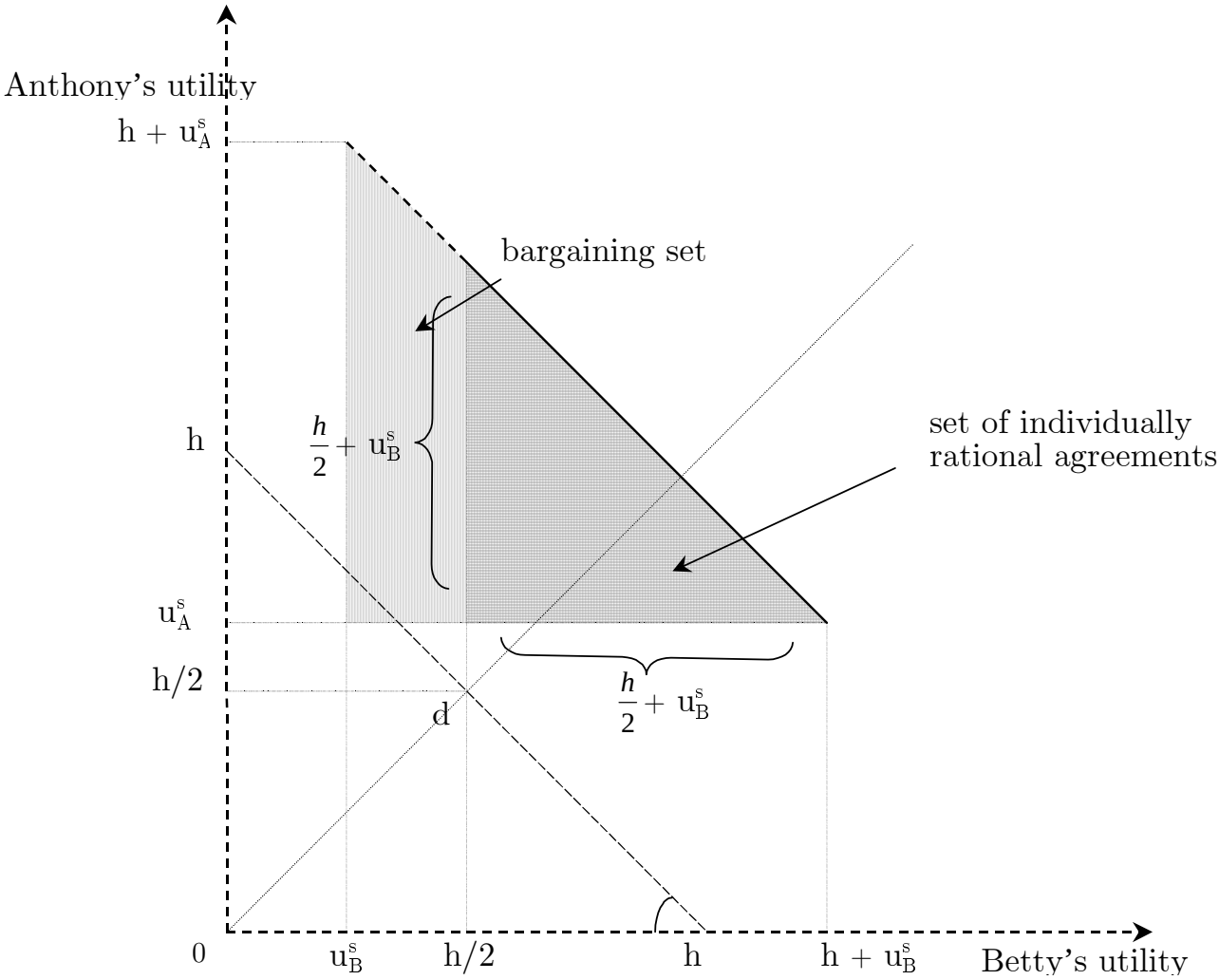
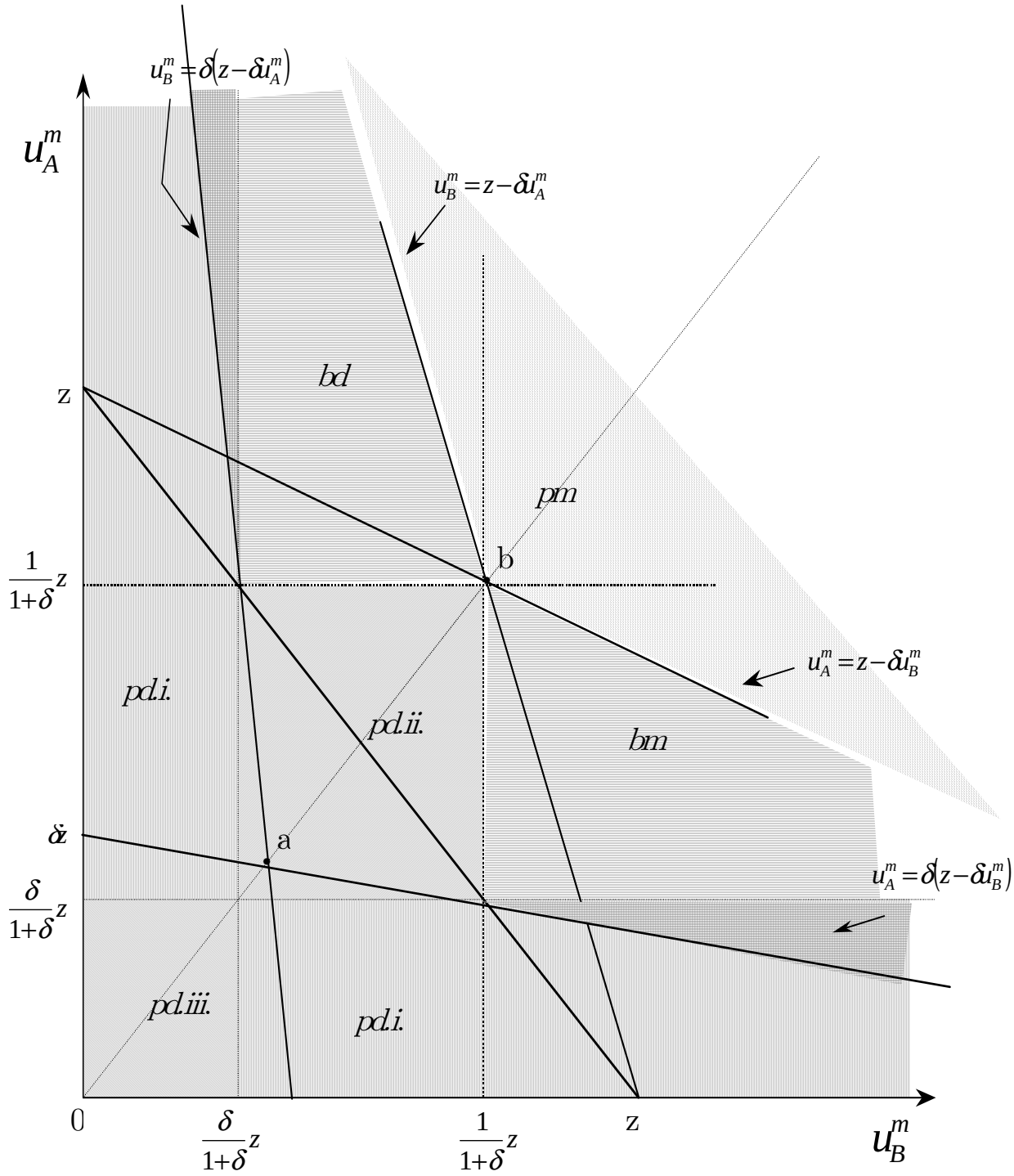


Figure 3: The case where $h/2 > u_i^s$ does not hold for all agents.



$$a \equiv \left(\frac{\delta}{1+\delta^2} z, \frac{\delta}{1+\delta^2} z \right)$$

Figure 4: Parameter configurations and stationary equilibria.

	Marriage		Divorce			
	(bm)*	(pm)	(pd.i)	(pd.ii)	(pd.iii)	(bd)*
player i:						
in divorce proposes	$\left(z-\delta u_j^m, \delta u_j^m\right)$	$\left(z-\delta u_j^m, \delta u_j^m\right)$	$\left(\frac{1}{1+\delta}z, \frac{\delta}{1+\delta}z\right)$	$\left(\frac{1}{1+\delta}z, \frac{\delta}{1+\delta}z\right)$	$\left(\frac{1}{1+\delta}z, \frac{\delta}{1+\delta}z\right)$	$\left(z-\delta u_j^m, \delta u_j^m\right)$
in divorce accepts	$x \geq \delta u_i^m$	$x \geq \delta u_i^m$	$x \geq \frac{\delta}{1+\delta}z$	$x \geq \frac{\delta}{1+\delta}z$	$x \geq \frac{\delta}{1+\delta}z$	$x \geq \delta u_i^m$
proposes to stay together	always	always	never	never	never	never
accepts to stay together	always	always	never	always	never	always
player j:						
in divorce proposes	$(z-\delta u_i^m, \delta u_i^m)$	$(z-\delta u_i^m, \delta u_i^m)$	$\left(\frac{1}{1+\delta}z, \frac{\delta}{1+\delta}z\right)$	$\left(\frac{1}{1+\delta}z, \frac{\delta}{1+\delta}z\right)$	$\left(\frac{1}{1+\delta}z, \frac{\delta}{1+\delta}z\right)$	$(z-\delta u_i^m, \delta u_i^m)$
in divorce accepts	$x \geq \delta u_j^m$	$x \geq \delta u_j^m$	$x \geq \frac{\delta}{1+\delta}z$	$x \geq \frac{\delta}{1+\delta}z$	$x \geq \frac{\delta}{1+\delta}z$	$x \geq \delta u_j^m$
proposes to stay together	never	always	never	never	never	always
accepts to stay together	always	always	always	always	never	always

*: assumes $i = B$, $j = A$.

Table 1: The equilibria of Proposition 1

Queen Mary
University of London

This working paper has been produced by
the Department of Economics at
Queen Mary, University of London

Department of Economics
Queen Mary, University of London
Mile End Road
London E1 4NS
Tel: +44 (0)20 7882 5096 or Fax: +44 (0)20 8983 3580
Email: j.conner@qmul.ac.uk
Website: www.econ.qmul.ac.uk/papers/wp.htm